Supersolid

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Supersolid

- "Superfluid-like solid"
- Intensively discussed by theorists in 1970s
- Still uncertain

Outline

- The pre-history of supersolid
  - Bose-Einstein Condensation(1924-25)
  - Superfluid(1937)
- Theoretical discussion
- Experimental search
  - Method
  - Result

Bose-Einstein Condensation

- Originally studied by Bose and Einstein
- Basic idea: Boson can be condensed into ground state

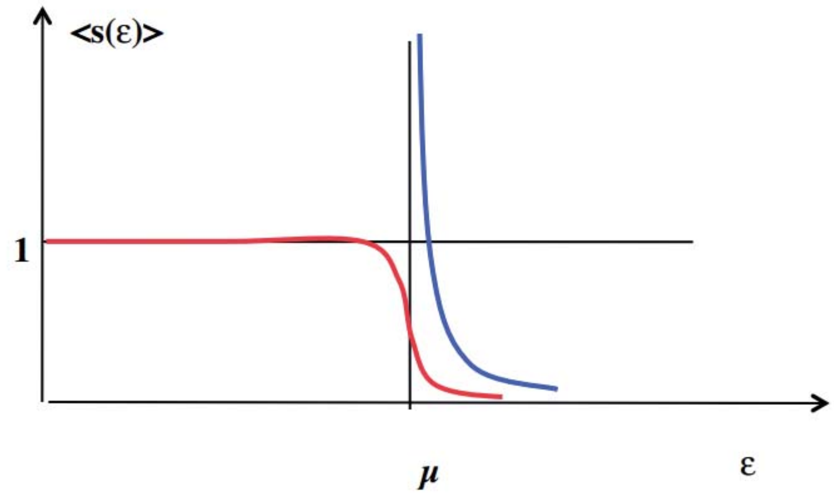
Bose-Einstein Condensation

: Fermi Dirac

$$\langle s(\varepsilon)\rangle = \frac{1}{exp\left(\frac{\varepsilon-\mu}{\tau}\right)+1}$$

Bose Einstein

$$\langle s(\varepsilon)\rangle = \frac{1}{\exp\left(\frac{\varepsilon-\mu}{\tau}\right)-1}$$

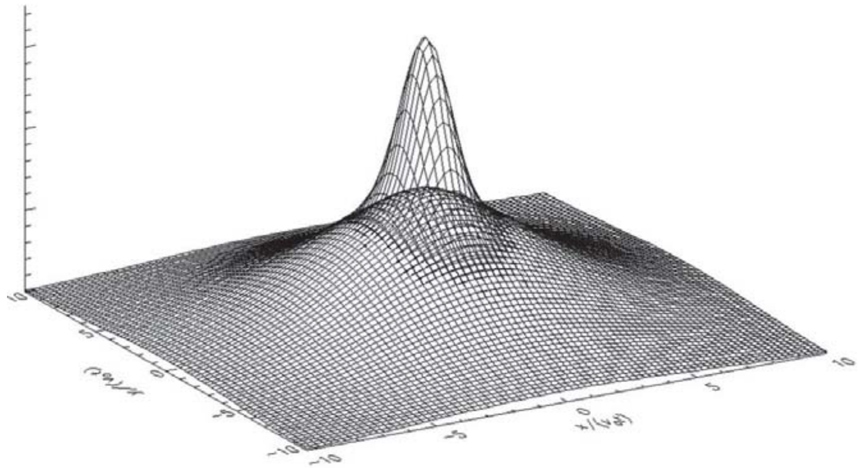


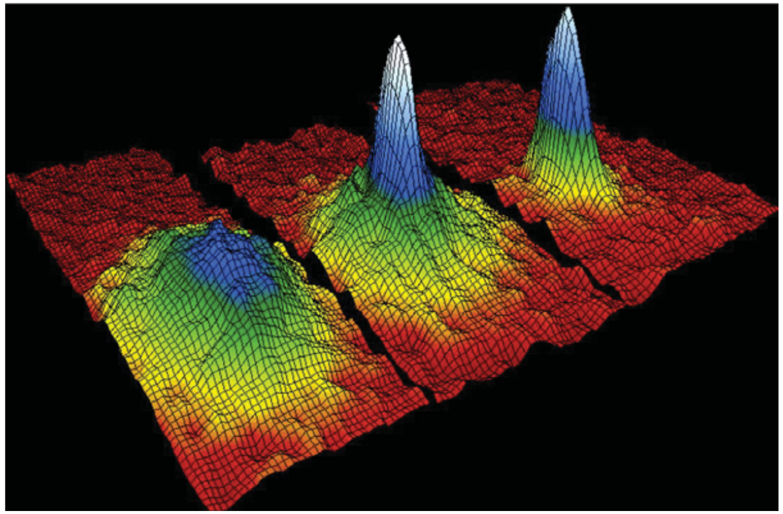
- From Phys 112 (S12), B.Sadoulet

Bose-Einstein Condensation

- Originally studied by Bose and Einstein.
- Basic idea: Boson can be 'squeezed' into ground state.
- Ideal gas

Bose-Einstein Condensation





- Anderson et al. (1995)
- Gas at low temperature for 'ideal gas' (^{87}Rb)
- http://www.colorado.edu/physics/2000/bec/index.html for more about this

Bose-Einstein Condensation

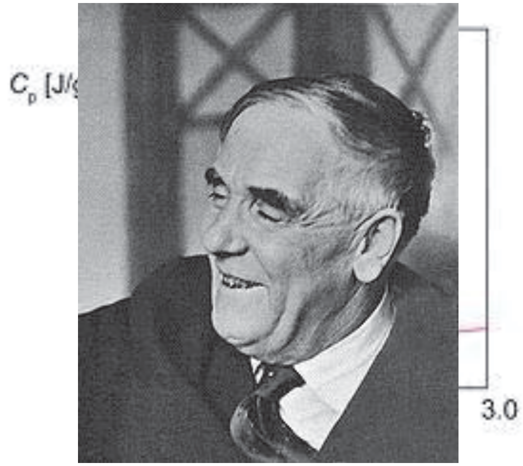
- Originally studied by Bose and Einstein.
- Basic idea: Boson can be 'squeezed' into ground state.
- Ideal gas:
  - No correlation between any two states
  - Nor among particles.
- What can be treated as ideal gas?
  - Cold gas
  - Anything else?

Superfluid

- Certain fluid can do it.
- Actually found long before gaseous BEC.
- Discovered and later explained.(1937)

Superfluid(^{4}He)

- Discovery: Pyotr Kapitsa, John F. Allen, and Don Misener
  - Abnormally high heat conductivity
  - Explanation: convection
  - Indicates very low viscosity
- Explained: London, Landau, etc.
  - Relating this phenomenon with BEC



Superfluid(^{4}He)

- Why this system can have BEC?

Bose-Einstein Condensation

- Originally studied by Bose and Einstein.
- Basic idea: Boson can be 'squeezed' into ground state.
- Ideal gas: (Analogy)
  - No correlation between states
  - Not modified by the presence of other particles.
- What can be treated as ideal gas?
  - Cold gas
  - Anything else?

Superfluid(^{4}He)

- Why He fluid analogy to ideal gas? Criteria?
- Feynman: Partition function (DOI:10.1103/PhysRev.91.1291)
- Bogoliubov: Momentum space

Superfluid(^{4}He)

- Bogoliubov's idea:
  - For particles without interaction:

$$\langle n_0 \rangle_{Avg}/N = e^{O(1)} \leftrightarrow \text{BEC}$$

$$\langle n_0 \rangle_{Avg}/N = O(1) \leftrightarrow \text{no BEC}$$

  - With interaction:

$$n_M/N = e^{O(1)} \leftrightarrow \text{BEC}$$

$$n_M/N = O(1) \leftrightarrow \text{no BEC}$$

Where n_M is the largest eigenvalue of σ_1, defined by

$$\sigma_1 = N tr_{2\ldots N}(\sigma)$$

Switching into momentum space:

$$\langle \mathbf{p_1}' | \sigma_1 | \mathbf{p_1}' \rangle = N \sum_{\mathbf{p_2}'} \cdots \sum_{\mathbf{p}_N'} \langle \mathbf{p_1}' \cdots \mathbf{p}_N' | \sigma | \mathbf{p_1}' \cdots p_N' \rangle$$

Superfluid(^{4}He)

- Why analogy to ideal gas? Criteria?
- Feynman: Partition function
  (DOI:10.1103/PhysRev.91.1291)
- Bogoliubov: Momentum space
- Penrose and Onsager

Supersolid?

- Can these be extended to happen in solid?
- O. Penrose and L. Onsager: 'proved' that superfluidity can only occur with fluid-like phase.(1956)

O. Penrose and L. Onsager

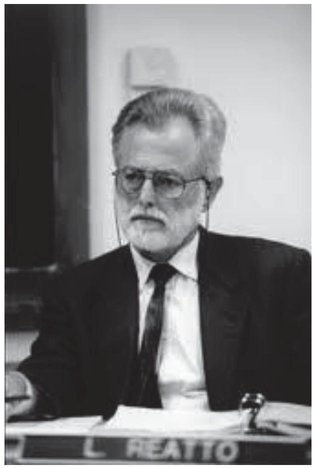
- Based on Bogoliubov's criteria
- Define weak interaction:
  - Finite R that limits correlation inside
- Lattice structure always leads to strong long range interaction

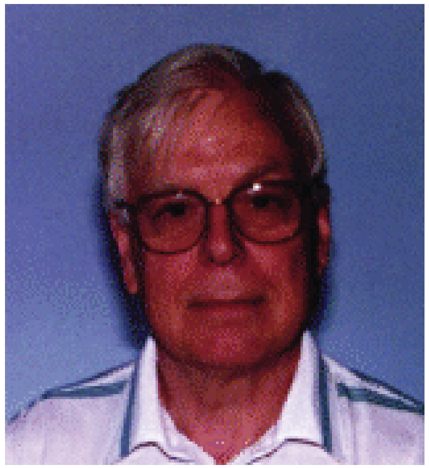
Supersolid?

- Can these be extended to happen in 'solid'?
- O. Penrose and L. Onsager: 'proved' that superfluidity can only occur with fluid-like phase.(1956)
- L. Reatto and G. V. Chester, Lifshitz, proposed a possiblity for superfluidity to appear simoutaneously with solid.

L.Reatto and G.V.Chester

- New theoretical progress:
  - Effective potential in lattice MAY decay fast enough
- Criteria: finite fraction of vacancy.





Difficulty

- Maybe exists, maybe not theoretically
- To make things worse, the property of 'supersolid' is not clearly predicted.

Experimental attempt to search for super solid

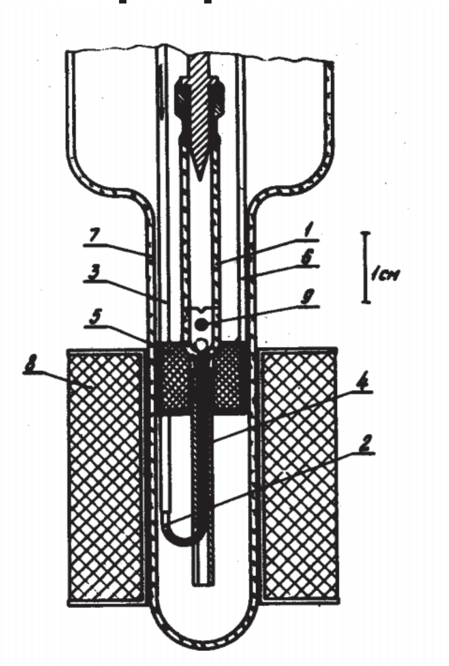
- Create supersolid
- Identify supersolid

First Attempt

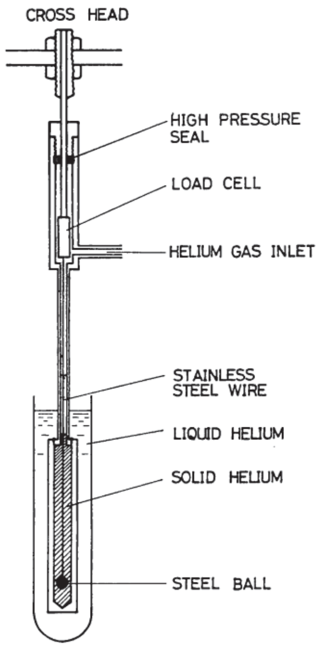
- Embed a ball in Helium crystal and measure the movement of the ball.
- A. Andreev et al. (1969)
- H. Suzuki (1973)
- V. L. Tsymbalenko (1976)
- Goal:
  - Check the premises: investigate vacancies.
  - See if there is non-classical movement

First Attempt

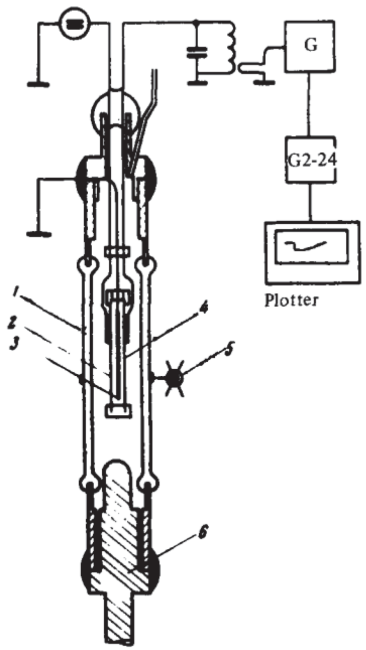
- Equipment



Andreev



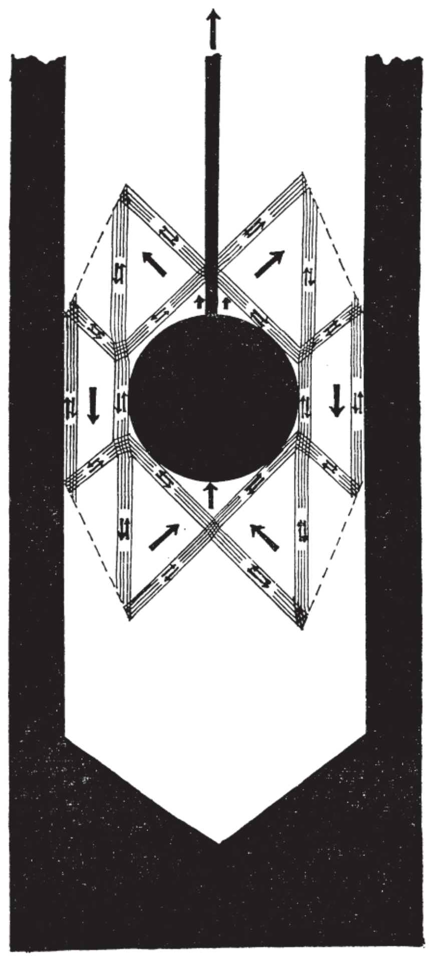
Suzuki



Tsymbalenko

First Attempt

- Investigate plastic flow
- Turn out to be the motion of dislocation
- Also, not so much vacancy



Further Attempt

- W. M. Saslow(1976) gives calculation indicating a fraction of 10^{-1} to be expected.
- D. S. Greywall(1977): Look for flow of He Directly.

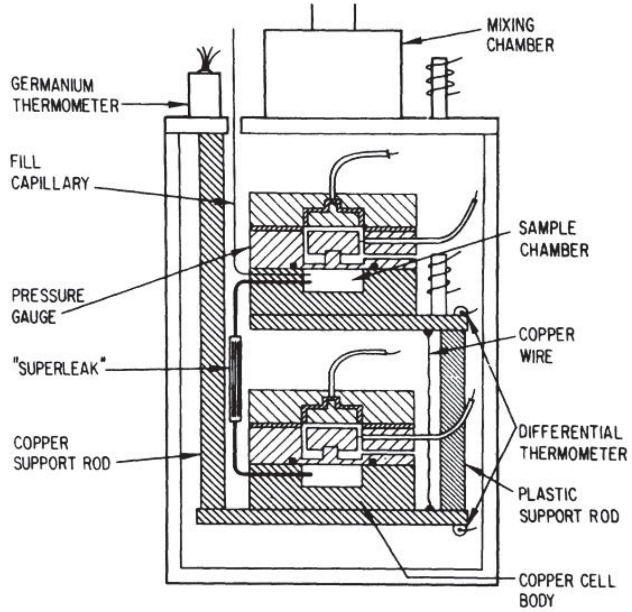
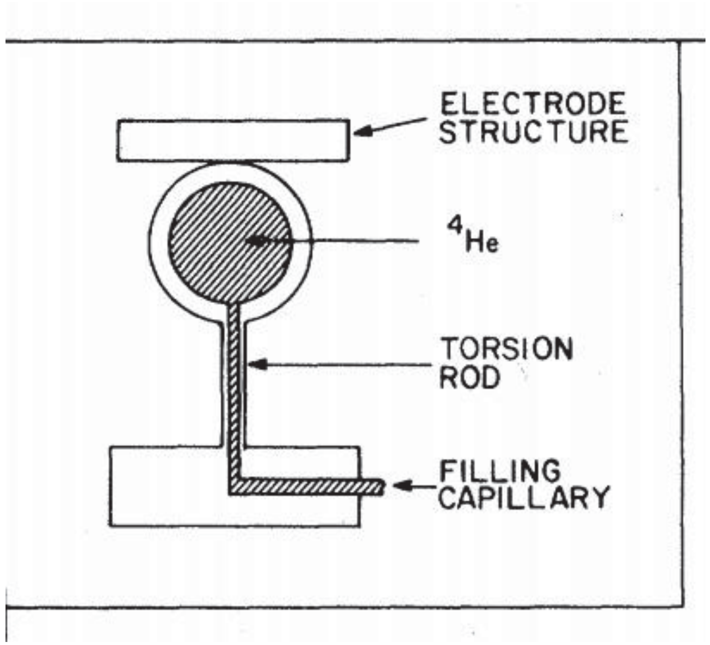


FIG. 1. Sample chambers.

Further Attempt

- A. J. Leggett (1970) suggests nonclassical rotational inertia.
- D. J. Bishop et al.(1981)
- Result: nothing non-classical
- 5×10^{-6} if exist



After 1981

- Gas BEC discovered in 1995
- But no significant progress in supersolid
- Until...

Some sort of success...

- E. Kim and M. H. W. Chan(2004)
- Claim a probable observation

letters to nature

Probable observation of a supersolid helium phase

E. Kim & M. H. W. Chan

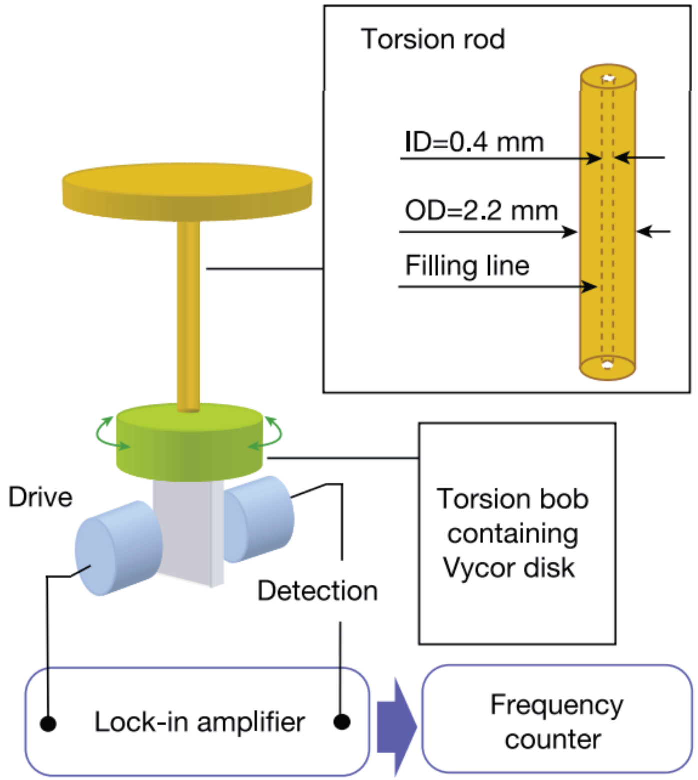
Department of Physics, The Pennsylvania State University, University Park, Pennsylvania 16802, USA

When liquid ^{4}He is cooled below 2.176 K, it undergoes a phase transition—Bose–Einstein condensation—and becomes a super-

high-Q oscillator shown in Fig. 1 is given by $2\pi\sqrt{I/G}$, where I is the moment of inertia of the torsion bob, which contains helium, and G is the torsional spring constant of the Be-Cu torsion rod. A small hole drilled through the centre of the torsion rod allows the introduction of helium into the torsion bob. The oscillator is driven and maintained at resonance by a pair of electrodes. The onset of superfluidity in the helium inside the torsion bob decreases I, and hence decreases the resonant period. Bishop *et al.*[8] made measurements of solid helium from 25 to 48 bar, and concluded that if there is a supersolid state, then either the supersolid fraction (the fraction of ^{4}He atoms participating in superflow) is less than 5×10^{-6} or the critical velocity is less than $5\ \mu m\ s^{-1}$. (The critical velocity is the maximum

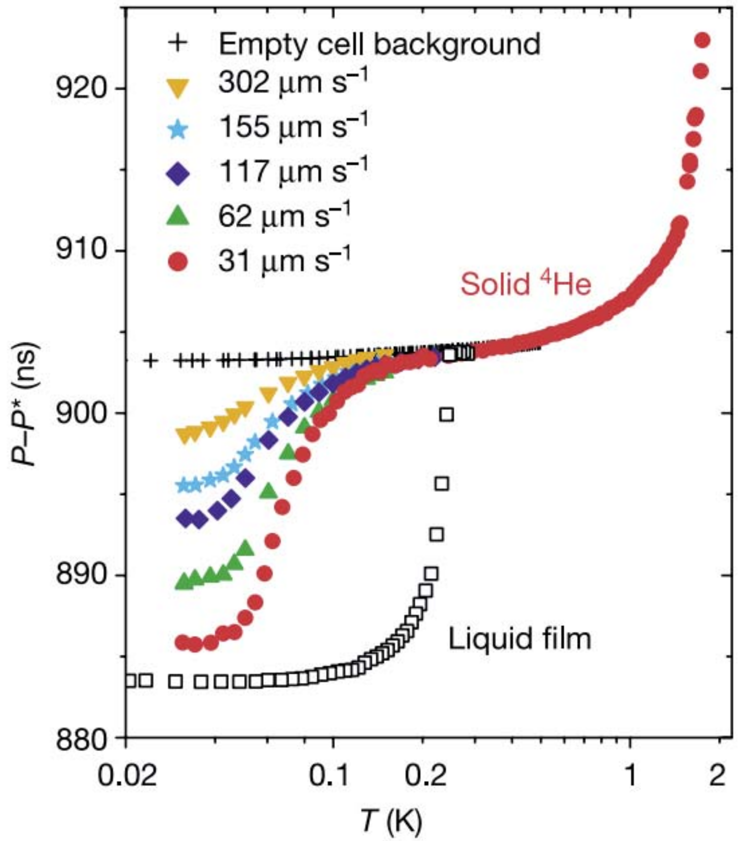
Some sort of success...

- Follows Bishop's method but improved in tech
- Vycor glass
- High Q

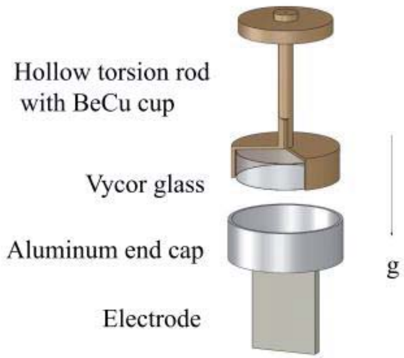


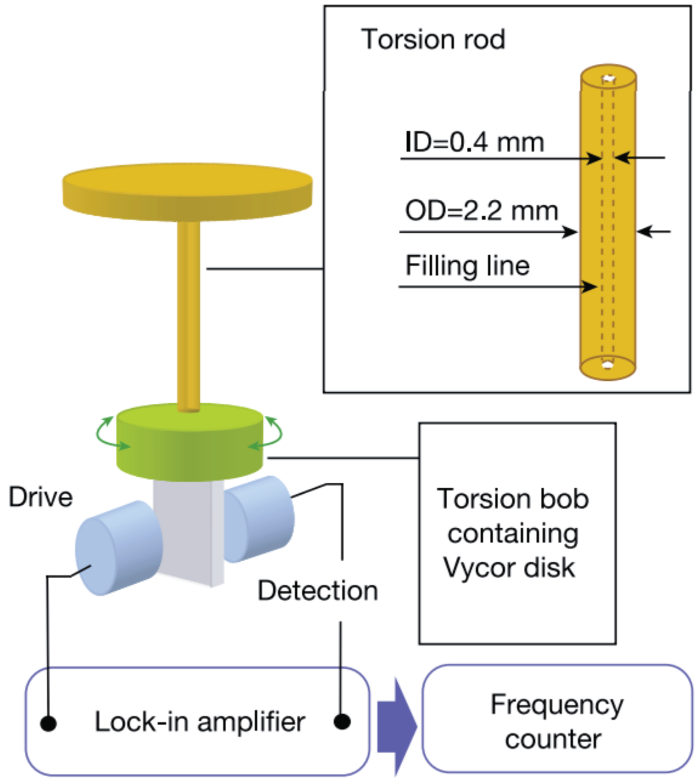
Some sort of success...

- Change in momentum of inertia
- Independent from solid-liquid phase change
- May be supersolid

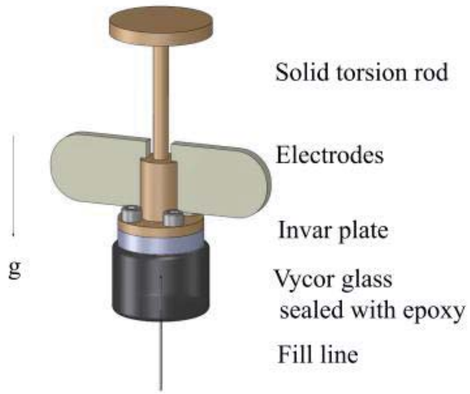


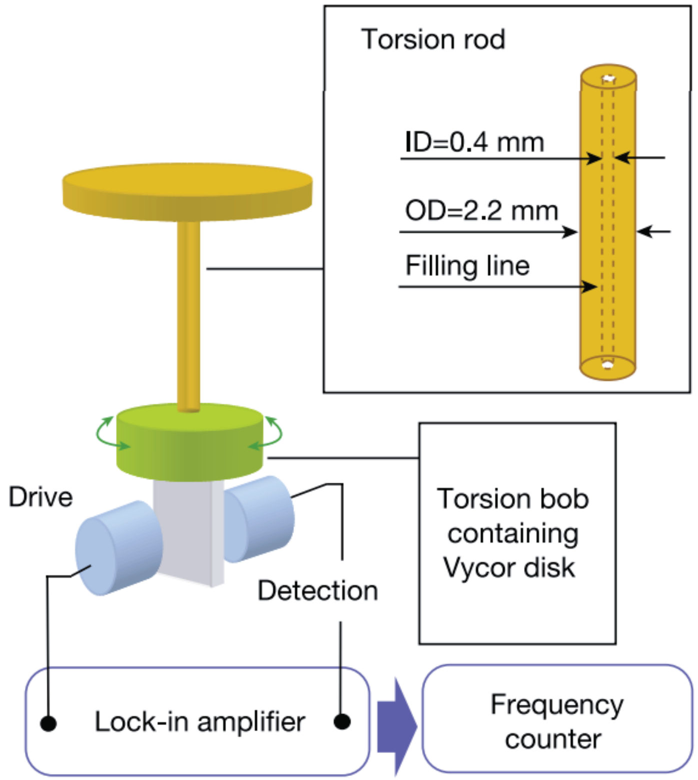
Some sort of success...





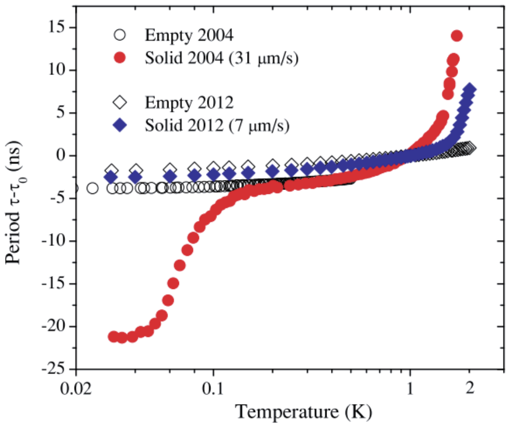
But

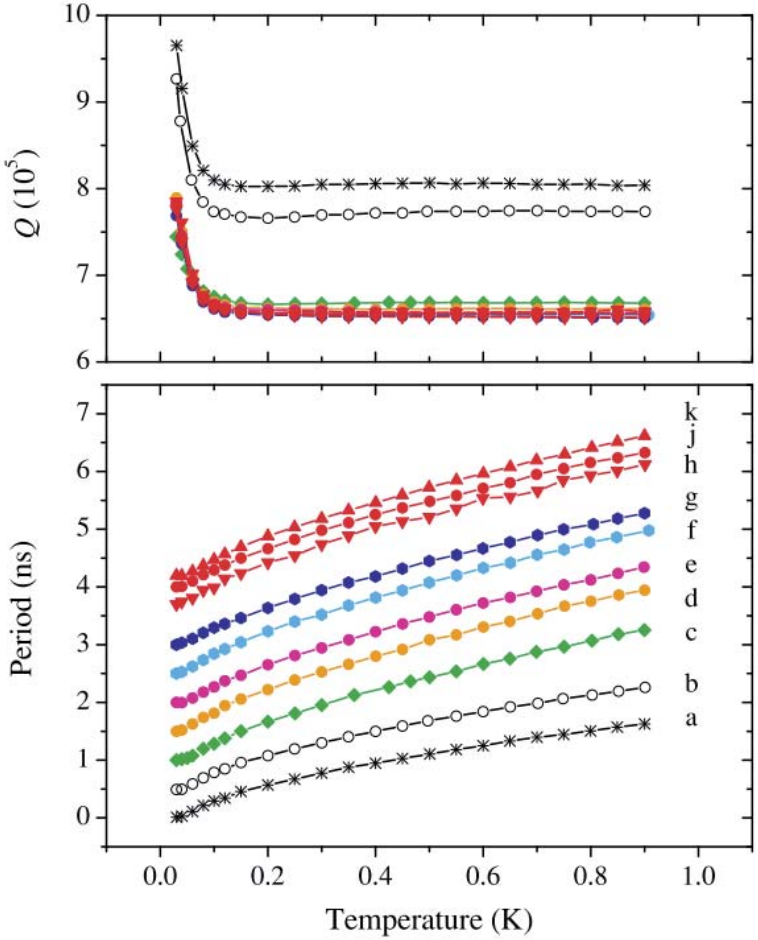

Solid torsion rod
Electrodes
Invar plate
g
Vycor glass
sealed with epoxy
Fill line


Torsion rod
ID=0.4 mm
OD=2.2 mm
Filling line
Drive
Detection
Torsion bob
containing
Vycor disk
Lock-in amplifier
Frequency
counter

But

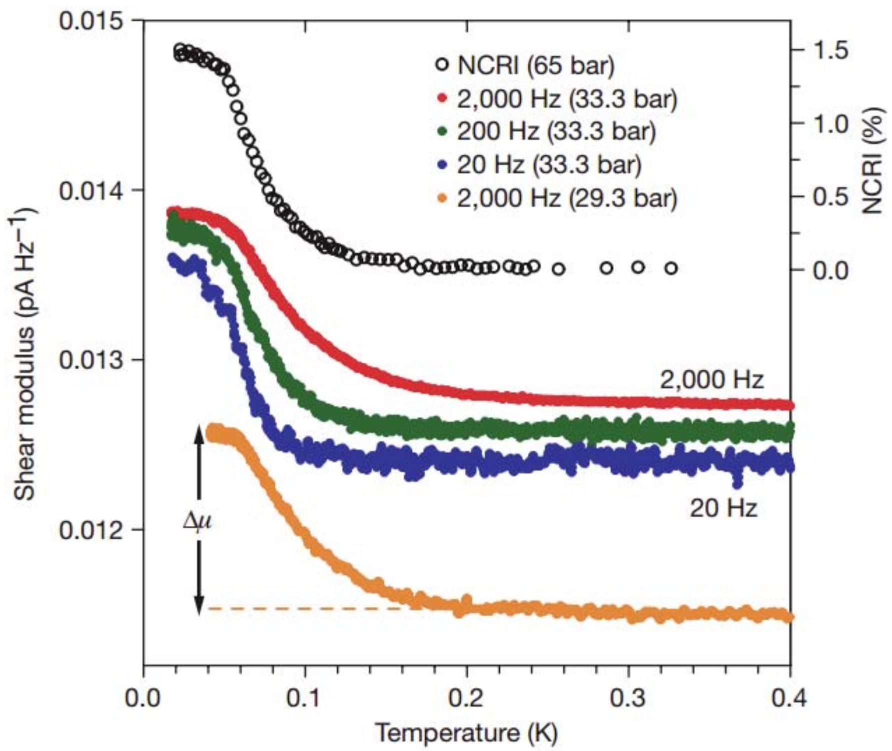
- Evidence swept out.
- The 'abnormal' produced by the equipment





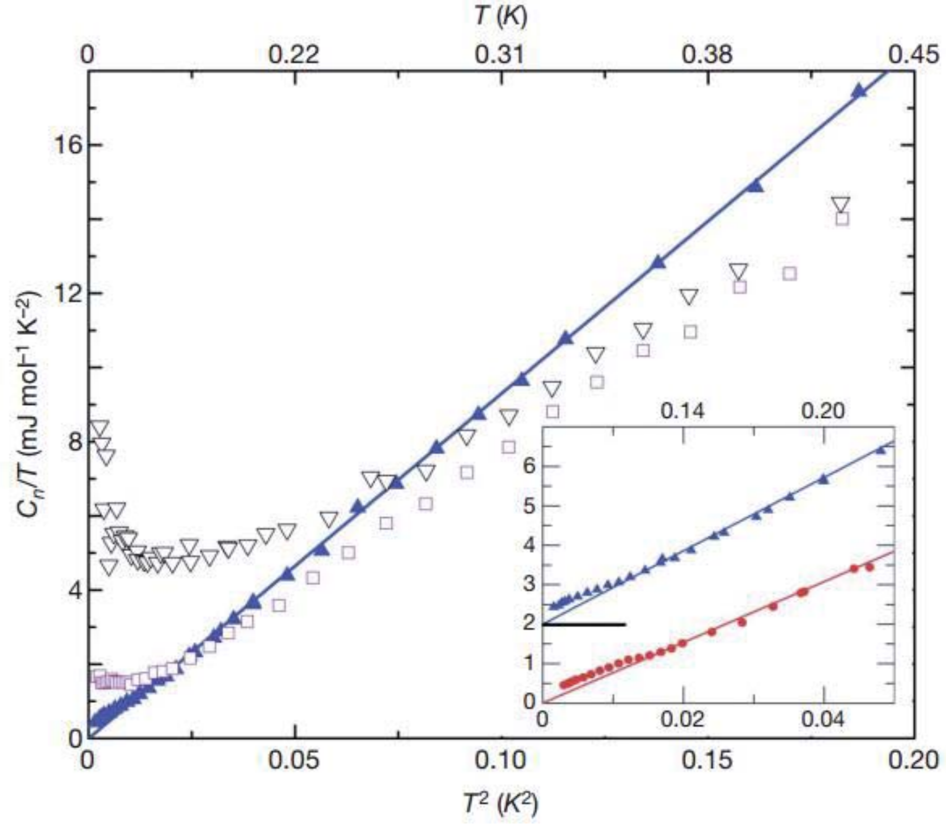
Other observations

- Shear modulus (2010)



Other observations

- Heat capacity peak(2007)



Open Future

- Theoretical prediction not swept out.
  - Except for the case of homogeneous crystal
- Ambiguity saves this phase, while producing a dilemma.

Why Concerned?

Refference-books & websites

- Statistical Mechanics, R.K.Pathria and Paul.D.Beale. ISBN 978-0-12-382188-1
- Bose–Einstein Condensation in Dilute Gases, C.J.Pethick and H.Smith. ISBN 0 521 66580 9
- http://www.colorado.edu/physics/2000/bec/index.html
- http://www.physicstoday.org/daily_edition/physics_update/where_did_the_supersolidity_go

Reference-papers(1)

- Viscosity of Liquid Helium below the λ-Point (1938)
- DOI:10.1038/141074a0

- Atomic Theory of the λ Transition in Helium (1953)
- DOI:10.1103/PhysRev.91.1291

- Bose-Einstein Condensation and Liquid Helium (1956)
- DOI:10.1103/PhysRev.104.576

- Attempt at Observing Vacancions[Vacancies] in 4He Crystals (1969)
- JETP Vol.9, pp.306

- Speculations on Bose-Einstein Condensation and Quantum Crystals (1970)
- DOI:10.1103/PhysRevA.2.256

- Plastic Flow in Solid Helium (1973)
- JPSJ Vol.35, No.5

- Plastic flow of crystalline 4He (1976)
- JETP Vol.23, pp.653 (Eng Vers.)

- Search for superfluidity in solid 4He (1977)
- DOI:10.1103/PhysRevB.16.1291

- Search for superfluidity in hcp 4He (1981)
- DOI:10.1103/PhysRevB.24.2844

- Observation of Bose-Einstein Condensation in a Dilute Atomic Vapor (1995)
- DOI:10.1126/science.269.5221.198

Reference-papers(2)

- Probable observation of a supersolid helium phase (2004)
- DOI:10.1038/nature02220

- What makes a crystal supersolid? (2006)
- DOI:10.1080/00018730601183025

- Probable heat capacity signature of the supersolid transition (2007)
- DOI:10.1038/nature06228

- Low-temperature shear modulus changes in solid 4He and connection to supersolidity (2007)
- DOI:10.1038/nature06383

- Intrinsic and dislocation-induced elastic behavior of solid helium (2009)
- DOI: 10.1103/PhysRevB.79.214524

- Dynamical Creation of a Supersolid in Asymmetric Mixtures of Bosons (2009)
- DOI: 10.1103/PhysRevLett.102.255304

- Supersolid behavior in confined geometry (2009)
- arXiv:0904.2373v1

- Glass Anomaly in the Shear Modulus of Solid 4He (2010)
- DOI:10.1103/PhysRevLett.105.045302

- Absence of supersolidity in solid helium in porous Vycor glass (2012)
- arXiv:1207.7050v1

- Plastic response of dislocation glide in solid helium under dc strain rate loading (2013)
- arXiv:1303.1852v1