Homework # 6 (the last one!); due Thursday, April 18

25. Electron-gas in a metal. In this problem, we will explore whether a classical free-gas model works for electrons in a metal.
    a. Assuming that each atom in a metal (let us say, copper, to be specific) donates one electron to the electron gas in bulk metal, what is the density of electrons in the gas?
    b. Assuming free non-interacting particles with electron mass, calculate the density of states as a function of kinetic energy of the electrons. (You probably want to start from the momentum or wave-vector space, and then go to energy.)
    c. Assume the metal is at zero temperature. Is it OK to forget that electrons are *fermions*? Motivate your answer.
    d. Calculate the *Fermi energy* of the gas. (Recall the definition of what it is first.) Give the answer in J, erg, eV, and K. Is it possible to practically achieve solid-metal temperature comparable to the Fermi energy?
    e. What is the Fermi velocity? Compare it to the speed of light.
    f. Assume now that the sample is at a finite temperature that is much smaller than the Fermi energy. Estimate the fraction of electrons that are occupying quantum states with occupation numbers not equal to unity.
    g. Estimate the *specific heat* of electron gas. How does it depend on temperature?
    h. At room temperature, do electrons make a significant contribution to the bulk heat capacity? At what temperature do you expect the electron contribution to dominate?

26. In class, we discussed the *dispersion relation* for electrons in a 1-D periodic potential:

$$\frac{\beta^2-\alpha^2}{2\alpha\beta}\sinh(\beta b)\sin(\alpha a)+\cosh(\beta b)\cos(\alpha a)=\cos[K(a+b)].$$

    a. Recall the definitions of all the quantities in the above expression.
    b. Simplify the dispersion relation for the case of $b \ll a \approx d$.
    c. Introducing a dimensionless parameter $P \equiv \frac{mV_o ba}{\hbar^2}$, consider two limiting cases: $P \to 0$ and $P \to \infty$. Please explain what's going on physically in each of these limits.

27. A dopant in a semiconductor (for example, an As atom is Si) can be thought of as an analog of a hydrogen atom once it substitutes for a Si atom in the lattice. In fact, As has an extra electron that is in excess of the four electrons that each silicon atom contributes that effectively "sees" the access positive charge of the As nucleus. In order to make the analogy quantitative, we need to account for the following differences between the As impurity and hydrogen: i) The effective mass m^* of the electron is different from the electron mass m ($m^* \sim 0.1\ m$), and ii) The dielectric constant of silicon ($\varepsilon \sim 10$) needs to be accounted for. Now to the problem:

a. Estimate the binding energy of the electron to the As site. Based on the result, explain why it is frequently said that the Fermi level of As doped Si lies "very close" to the conduction band.
b. Estimate the "first Bohr radius" for this system. Based on the result, justify the need to account for the dielectric constant of the medium (we do not talk about anything like that when we calculate the energy of free hydrogen atoms).
c. Arsenic doped silicon is called *n-type semiconductor*. What would be a proper dopant to make silicon a *p-type semiconductor* (*n* and *p* refer to the negative and positive charge of access carriers, respectively)?

28. Consider a defect in a crystalline lattice of an *alkali halide* (e.g., NaCl), where one of the halogen atoms is missing. Such a *vacancy* corresponds to a *color center* (a.k.a. *F-center*), and is responsible for optical absorption of the crystal. Assuming that F-center's electronic energy levels are those of an "electron in a box,"
    a. Verify that the order of magnitude of the excitation energy indeed corresponds to optical absorption;
    b. Find the dependence of the electron-excitation energy on the distance *a* between adjacent alkali and halogen atoms.