Homework # 3; due Thursday, March 7

12. This problem will illustrate the basic physics of exchange interaction. Let us consider the expectation value $\langle (x_2 - x_1)^2 \rangle$ for two electrons in a 1D simple harmonic-oscillator (SHO) potential, where x_1 and x_2 are the positions of the two electrons. For the case where one of the electrons is in the ground state and the other is in the first excited state, calculate the above expectation value for the triplet and singlet spin states. The following representation may be useful:

$$\hat{x}_i = \sqrt{\frac{\hbar}{2m\omega}}\left(\hat{a}_i + \hat{a}_i^\dagger\right),$$

$$\hat{a}_i |n\rangle_i = \sqrt{n}\,|n-1\rangle_i\,,$$
$$\hat{a}_i^\dagger |n\rangle_i = \sqrt{n+1}\,|n+1\rangle_i\,.$$

Here we use the standard notations for the creation and annihilation operators and SHO states.

13. Show that the angle between any two of the lines (bonds) joining a site of the diamond lattice to its four nearest neighbors is 109°28'.

14. One of many known defects in diamond crystals is a *vacancy*, i.e., a missing carbon atom in the diamond lattice. It has been determined that a vacancy can be formed if one imparts a kinetic energy of 40 eV (or more) to a carbon atom in the lattice, thus "kicking" the atom out of the lattice.
    a. Estimate the minimum energy of an electron beam that is needed to produce the vacancies. Is it important to include relativistic effects in this estimate?
    b. Suppose we need to produce uniformly distributed vacancies in a diamond plate which is 1 mm thick using a beam of electrons. What energy electrons should be used?
    c. In practice, researchers tend to avoid using electrons with energies in excess of 15 MeV or so for this purpose. Can you think of a reason why?

15. The energy splitting between the M=0 and M=±1 ground-state spin sublevels of the NV^- color center in diamond corresponds to a frequency of about 2.9 GHz. Assuming that the center is in thermal equilibrium with the environment, to what temperature should one cool the sample so that 90% of the population is in the lower (M=0) sublevel?