Introduction to Nuclear Physics

Physics 124

Solution Set 5

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Problem 18

a) The total cross section for Coulomb scattering should give you an infinite result when you perform the integration. This is due to the fact that the electromagnetic force is mediated by photons which have an infinite range. In other words, even an electron 1000 meters away from a nucleus will be scattered by a small angle. However in any real physical system the Coulomb potential is screened by the electrons surrounding a nucleus. Therefore the range of the scattering potential is limited to less than a few Angstroms.

b) For this question we make use of classical mechanics and appropriate approximations to determine the energy loss of a charged particle passing through matter. First of all we assume that the dominant energy loss is through ionization of the target material. Therefore we are concerned only with the momentum transferred to the electrons of the target material. For a given electron, we have:

$$P = F\delta t = \frac{Ze^2}{b^2} \times \frac{2b}{v} = \frac{2Ze^2}{bv}. \tag{1}$$

Here we have defined the following parameters:

P = momentum

F = force

δt = interaction time

Ze = charge of incoming particle

e = charge of electron

b = impact parameter

v = velocity of the incoming particle

The energy lost by the incoming particle can then be written as

$$-\delta E = \frac{P^2}{2m} = \frac{2Z^2e^4}{mb^2v^2}, \tag{2}$$

where m equals the mass of the scattered electron.

Now, suppose our particle travels a distance δx in the medium. How many electrons are there in a tube of volume,$2\pi b\delta b\delta x$? The number of electrons in the volume is given by

$$2\pi b\delta b\delta x \times n[\frac{atoms}{cm^3}] \times Z, \tag{3}$$

where n is the density of atoms and Z is the number of electrons per atom. n can also be represented by the following

$$n = \frac{\rho(\frac{g}{cm^3})}{A(\frac{g}{mole})} \times 6.02 \times 10^{23} \frac{atoms}{mole} \tag{4}$$

Here A is the atomic number of the material.

Now we can calculate the energy lost per distance traveled. The only thing we must remember is to integrate over the impact parameter to take into account all possible impact parameters. Therefore, the energy loss of the particle per unit distance traveled is given by:

$$-\frac{dE}{dx} = 2\pi \int (\frac{2Z^2e^4}{mb^2v^2}) nZbdb \tag{5}$$

$$-\frac{dE}{dx} = 2\pi \int (\frac{2Z^2e^4}{mb^2v^2}) \frac{\rho(\frac{g}{cm^3})}{A(\frac{g}{mole})} 6.02 \times 10^{23} \frac{atoms}{mole} Zbdb \tag{6}$$

We can say, for our calculation, that $\frac{Z}{A} = \frac{1}{2}$ and also use the definition $\beta = \frac{v}{c}$.

$$-\frac{dE}{dx} = 2\pi (\frac{Z^2e^4}{mc^2\beta^2}) \rho(\frac{g}{cm^3}) 6.02 \times 10^{23} \int_{b_{min}}^{b^{max}} \frac{db}{b} \tag{7}$$

Evaluating the constants and setting $b_{min} = 10^{-13}$ cm (nuclear diameter) , $b_{max} = 5 \times 10^{-9}$ cm (typical atomic radius beyond which there is no overall Coulomb field from an atom) we obtain

$$-\frac{dE}{dx} = 2\frac{Z^2}{\beta^2}\rho[\frac{g}{cm^3}], \tag{8}$$

in $[\frac{Mev}{cm}]$. Note that the result is quite insensitive to the exact choice of the integration limits because the integral is a logarithm.

Problem 19

A thermal neutron is taken to have an energy of $\frac{1}{40}$ eV, which corresponds to room temperature (300 K). The velocity of the neutron is simply related to its thermal energy by

$$E = \frac{3}{2}kT = \frac{1}{2}mv^2. \tag{9}$$

The only thing one has to be careful of is conversions between units. Let's list some common values:

Boltzmann constant, k = $1.38066 \times 10^{-23}\frac{J}{K} = 8.6174 \times 10^{-5}\frac{eV}{K}$

Neutron mass, m = 1.00866501 amu = 1.67495482 $\times 10^{-27}$ kg

Planck's constant, h = 6.62618 $\times 10^{-34}$ J-s = 4.13570 $\times 10^{-15}$ eV-s

Conversion: 1 eV = 1.602189 $\times 10^{-19}$ J

From here you can proceed a number of different ways. Here is the most obvious way to solve the problem.

a) Solve for v in terms of mks units therefore the only conversion you need is between eV and Joules.

$$v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2 \times \frac{1eV}{40} \times 1.602 \times 10^{-19}\frac{J}{eV}}{1.6749 \times 10^{-27}kg}} = 2 \times 10^3 \frac{m}{s} \tag{10}$$

b) Solve for the deBroglie wavelength using the velocity value obtained in part a) and the neutron mass and Planck's constant in mks units.

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.626 \times 10^{-34}Js}{(1.67495 \times 10^{-27}kg)(2191.7\frac{m}{s})} = 2 \times 10^{-10}m \tag{11}$$

Problem 20

In this problem we are trying to make a source of polarized Ultra Cold Neutrons (UCN). This is actually done at Los Alamos National Laboratory using a more sophisticated technique, but with essentially the same physics as this problem describes.

a) Neglecting edge effects means considering the neutrons impinging on an infinite plate. What is the B-field outside the plate ?

The magnetic field inside the plate is created by oppositely directed surface currents on the top and bottom surfaces of the plate. Because the plate is infinite, the fields created by these currents are uniform. From the right hand rule, it is easy to see that the contributions of the two surfaces add inside the plate and subtract outside.

Therefore B = 0 outside the plate.

b) In order to reflect an incoming UCN we must understand how it interacts with the magnetic field. This is interaction is given by the following Hamiltonian

$$E = -\vec{\mu_n} \cdot \vec{B}. \tag{12}$$

Since we know the energy of the incoming UCN we need only compute the magnet field for our polarizer

$$B = \frac{E}{\mu_n} = \frac{10^{-7} eV}{1.913 \times 3.15245 \times 10^{-8} \frac{eV}{T}} \approx 1.7 Tesla. \tag{13}$$

This is the minimum magnetic field needed to reflect UCN with the wrong polarization.

Problem 21

Gravity attracts UCN like any other massive object. So how big of an effect is it on an UCN? Well we can calculate the energy gained by allowing the UCN to fall through a given distance of the gravitational field of the Earth, e.g. 1 meter.

$$U = mgh = (1.67495\times10^{-27}kg)(9.81\frac{m}{s^2})(1m) = 1.643\times10^{-26}J = 1.03\times10^{-7}eV \tag{14}$$

So we can see that the energy gained by an UCN in falling one meter is equivalent to doubling its energy. The argument works in reverse as well. Imagine you produce UCN that have a velocity away from the center of the Earth, they will essentially come to rest after a meter of travel.