Introduction to Nuclear Physics

Physics 124

Solution Set 1

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September 3, 1999

Problem 1

In this problem we are interested in calculating the energy released from the nuclear reactions in part a) and b). In order to do this we must first understand the basics of nuclear binding energy. From Einstein's equation of mass and energy we have the following relation:

$$E^2 = m^2c^4 + p^2c^2. \tag{1}$$

We take the common view point that the nuclear reaction occurs at zero momentum because we are only concerned with the rest mass energy change for binding energy calculations. Therefore we write:

$$E = mc^2. \tag{2}$$

Now we want to equate the energy of the system of particles before and after the nuclear reaction taking care to remember the electrons associated with the nuclei involved. (note: you rarely find fully ionized atoms in nature, except in stars.)

$$m_{nu}c^2 = m_A c^2 - Zm_e c^2 + \sum_{i=1}^{Z} B_i \tag{3}$$

m_{nu} = the mass of the nuclide after the reaction
m_A = atomic mass of the nuclide before the reaction
Zm_e = is the number of protons times the mass of an electron, accounting for the electron masses
B_i = is the binding energy of the electrons to the nucleus

We usually ignore the binding energy of the electrons to the nucleus. This is due to the relatively weak binding of the electrons compared to the nuclear binding energy scale and the electron mass.For example atomic electrons are bound with an energy on the order of a few eV up to 10 to 100 keV for the inner most electrons in larger atoms. Typical nuclear binding energies are on the order of a few MeV and are therefore 10 to 100 times greater. So we can write our working equation as

$$m_{nu}c^2 = m_A c^2 - Zm_e c^2. \tag{4}$$

Now what is the equation for the released energy? Simply take the energy of the particles that individually make up the nucleus and subtract the energy of the nucleus that is formed:

$$E_{rel} = \sum m(initialparticles)c^2 - \sum m(finalparticles)c^2. \tag{5}$$

Let's solve these problems:

a)

$$^9Be +^4 He \rightarrow^{12} C + n \tag{6}$$

In the back your nuclear physics book you can look up the atomic masses.
$m(^9Be)$ = 9.012182 u
$m(^4He)$ = 4.002603 u
$m(^{12}C)$ = 12.000000 u
m_n = 1.00866501 u
So we use the above equation to calculate the released energy as:
E$_{rel}$ = (m(^9Be)c^2 + m(^4He)c^2) - (m(^{12}C)c^2 + m(n)c^2)
E$_{rel}$ = (6.11999 $\times$10$^{-3}u)c^2$ = 5.701 MeV

b)

$$^7Li +^1 H \rightarrow^4 He +^4 He \tag{7}$$

$m(^7Li)$ = 7.016003 u
$m(^1H)$ = 1.007825 u
$m(^4He)$ = 4.002603 u
And again we use the above equation to calculate the binding energy as follows:
E$_{rel}$ = (m(^7Li)c^2 + m(^1H)c^2) - 2m(^4He)c^2,
E$_{rel}$ = (1.86220 $\times$10$^{-2}u)c^2$ = 17.346 MeV.

Problem 2

We have to consider why ^8Li is unstable by examining the mass data. Well if something is unstable it will decay to a more stable configuration and liberate energy. Lithium-8 is composed of 3 protons and 5 neutrons. We know (at least some of us know) that baryon number is a conserved quantity (from particle physics). A baryon is any particle which consists of three quarks. Such as a proton (uud quarks) and a neutron (udd quarks). In other words if you start with 8 protons and neutrons you should finish with 8 protons and neutrons. However a neutron could become a proton (β^- decay) or a proton could become a neutron (β^+ decay). So the two cases to consider are:
1) Z-1, N+1 a proton becomes a neutron
2) Z+1, N-1 a neutron becomes a proton

case 1)
Z = 2 and N = 6 this would imply that we form ^4He + 4n
$^8Li \rightarrow ^4He + 4n$
E$_{rel}$ $= m(^8Li)c^2 - m(^4He)c^2 - 4m(n)c^2$
E$_{rel}$ = -13.76 MeV

case 2)
Z = 4 and N = 4 this is beryllium-8
$^8Li \rightarrow ^8Be$
E$_{rel}$ $= m(^8Li)c^2 - m(^8Be)c^2$
E$_{rel}$ = (8.022486u)c^2 - (8.005305u)c^2 = 16.00 MeV

So we see that case 1 is not energetically favorable, but case 2 could occur and liberate 16 MeV per event. Note that stable nuclei exist for $1 \leq$ A $\leq$ 255 with the exception of A = 5 and A = 8. In this problem we have shown explicitly why one of the A = 8 nuclei is unstable.

Problem 3

I will be very open minded about what a correct answer is. However your logic has to be sound.

a) physicist = a person with a PhD in physics

Berkeley graduates about 20 or so PhDs per year. Let's say there are about:

70 PhD granting universities in North America

20 in South America

25 in Africa

80 in Asia

100 in Europe

10 in Australia

305 total

If each graduates 20 per year.

If each physicist works for 40 years.

20 x 40 x 305 = 244,000 physicists worldwide

This sounds like a lot however it represents 0.004% of the world's population.

b) weight of the Companile (in grams)

Let's say the average density is equal to that of water 1.0 grams/cm^3. Now all we have to do is determine the volume. The base is about 30 feet by 30 feet and its about 120 feet tall. The volume and mass are then found to be:

$V = 30 feet \times 30 feet \times 120 feet \times (\frac{30cm}{1 foot})^3 = 2.9 \times 10^9 cm^3$,

$Mass = (1.0 \frac{grams}{cm^3})(2.9 \times 10^9 cm^3) = 2.9 \times 10^9 grams.$

Even though my estimates on the height (actually over 200 feet high) and the base dimension (actually around 36 feet by 36 feet) are a bit off, I am within a factor of 2 of the actual weight which is around $4.5 \times 10^9 grams$ I went to check my answer by actually going for a ride to the top and looking at the construction plans.

c) cucumbers eaten by a person in their lifetime

First lets say that we are dealing with people who actually eat cucumbers. If your allergic then you probably ate one in your lifetime. How else do you know you have the allergy? Anyway pickles are cucumbers as well. So let's guess that you eat about one every three months in salads, on hamburgers, or simply cucumbers as a snack. From here its easy: cucumbers eaten = 80 years $\times$ 4 per year = 320 cucumbers, anywhere in that range will do.

d) height of the Earth's atmosphere

The lower limit is you know that jet liners fly at about an altitude of 30,000 feet which is about 9 kilometers. Another approach is to make use of atmospheric pressure at sea level. Say the pressure is about 1 $\frac{gram}{cm^2}$. Assuming a uniform gas column the force can be written as $mg = A\rho h$ and the density (ρ) is given by:

$$\rho = \frac{28grams}{22.4liters} = \frac{28grams}{22.4liters\frac{1000cm^3}{1liter}} = 0.00125\frac{grams}{cm^3}. \quad (8)$$

So the height turns out to be 10 km.

e) electric and magnetic fields on the surface of a nucleus

Use Gauss's law and the radius of a nucleus to estimate the electric field

$$\int EdA = \frac{Q}{\epsilon_o}. \quad (9)$$

Let's pick a worth while nucleus Z = 50 and N = 50. So A = Z + N = 100 and therefore $R = 1.2fm \times (100)^{\frac{1}{3}}$. Then we can calculate the electric field:

$$E = \frac{eZ}{4\pi\epsilon_o R^2} = 2 \times 10^{21}\frac{V}{m}. \quad (10)$$

Roughly speaking the magnitudes of E and B are related by E=cB. Therefore The magnitude of B = 7×10^{12} Telsa.

Problem 4

This problem required a little more attention than the others. You can determine the Coulomb repulsion energy by first determining the electrostatic energy of the system and then comparing the two cases Z to Z+1.

The electrostatic energy of the system is given by (Griffiths, Electrodynamics, Chap 2.4)

$$W = \frac{\epsilon_o}{2}\int_{allspace} E^2 d\tau. \tag{11}$$

You must take care to integrate over the correct electric fields. They are different inside and outside the nucleus. Using Gauss's law and spherical symmetry we can find the electric fields.

Inside the charge is distributed as

$$Q = \frac{eZ\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3} = \frac{eZr^3}{R^3}. \tag{12}$$

Therefore the electric fields inside the nuclear radius and outside the nuclear radius are found to be:

$$E_{inside} = \frac{eZr}{4\pi\epsilon_o R^3}, \tag{13}$$

$$E_{outside} = \frac{eZ}{4\pi\epsilon_o r^2}. \tag{14}$$

So in order to solve for all space we must break the integral up at the boundary of the nuclear radius:

$$W = \frac{\epsilon_o}{2}\Big(\int_{inside} E^2_{inside} d\tau + \int_{outside} E^2_{outside}\Big), \tag{15}$$

$$\int_{inside} E^2_{inside} d\tau = \int_0^R \frac{Z^2e^2r^2}{(4\pi\epsilon_o)^2 R^6} r^2 4\pi dr = \frac{Z^2e^2}{4\pi\epsilon_o^2}\frac{1}{5R}, \tag{16}$$

$$\int_{outside} E^2_{outside} = \int_R^\infty \frac{Z^2e^2}{(4\pi\epsilon_o)^2 r^4} r^2 4\pi dr = \frac{Z^2e^2}{4\pi\epsilon_o^2}\frac{1}{R}, \tag{17}$$

$$W = \frac{Z^2e^2}{(2)4\pi\epsilon_o}\Big(\frac{1}{5R} + \frac{1}{R}\Big) = \frac{Z^2e^2}{(2)4\pi\epsilon_o}\frac{6}{5R}. \tag{18}$$

In cgs (Gaussian units) $\epsilon_o = \frac{1}{4\pi}$ so we have

$$W = \frac{Z^2 e^2}{2}\frac{6}{5R}. \tag{19}$$

Now we need to calculate the difference in energy when a neutron becomes a proton in the nucleus:

$$\delta W = \frac{(Z+1)^2 e^2}{2}\frac{6}{5R} - \frac{Z^2 e^2}{2}\frac{6}{5R}, \tag{20}$$

$$\delta W = \frac{6e^2}{(2)5R}\Big(Z^2 + 2Z + 1 - Z^2\Big), \tag{21}$$

$$\delta W = \frac{6e^2}{(2)5R}\Big(2Z + 1\Big), \tag{22}$$

$$\delta W = \frac{6Ze^2}{5R} + \frac{3e^2}{5R}. \tag{23}$$

You can ignore the second term for Z $\gg$ 1. However if Z is very small, on the order of two or three, you should probably include it. Remember that Z here refers to the number of protons present before a neutron was converted to a proton.

For our problem A = 15 and Z = 7. We can then calculate the radius R and the energy difference. Using the mks solution, put back the $4\pi\epsilon_o$

$$R = 1.2fm(15)^{\frac{1}{3}} = 2.96fm, \tag{24}$$

$$\delta W = \frac{6(7)(1.602 \times 10^{-19} Coul)^2}{5(2.96fm)4\pi(8.85 \times 10^{-12}\frac{Coul^2}{Nm^2})} = 6.55 \times 10^{-13} Joules = 4.09 MeV. \tag{25}$$

Note that the second term is equal to 0.292 MeV which represents a 7% correction for Z = 7.