Homework # 9; due Thursday, November 11

Reading: Chapters 6, 7, and 9 of Griffiths

31. Paul Dirac was bothered by the question: why is that the position and momentum of a particle seem to be continuous, but the charge appears to be quantized. He was also bothered by the apparent asymmetry of the Maxwell equations due to the absence of magnetic monopoles. He considered the consequences of a possible existence of a magnetic monopole and derived a relation between the electric charge of the electron, \(-e\), and the magnetic charge of the monopole, \(g\). You will derive this relation in this problem.

Consider a non-relativistic electron scattering from a stationary and very heavy monopole. We first work in the classical-physics approximation. Assume that the impact parameter is \(b\), and that the trajectory, to the lowest approximation, is just a straight line.

a. Calculate the magnetic field acting on the electron as a function of time.
b. Calculate the corresponding Lorentz force. What is the direction of this force?
c. Calculate the angular momentum transferred to the electron in the course of the scattering.

We now bring in Quantum Mechanics (QM).

d. According to QM, orbital angular momentum is “quantized in units of \(\hbar\).” Write down this quantization condition. This is the famous Dirac relation.

It is sometimes said that the quantization of the electrical charge follows from the existence of just a single magnetic monopole somewhere in the Universe. What you have just derived equips you for pondering this assertion…

32. What is the relation between the differential cross section \(d\sigma/\ d\Omega\) and the total cross-section in the case of isotropic scattering? (Isotropic scattering is when the differential cross-section does not depend on either of the angles \(\theta\) or \(\varphi\).) Give an example of a process where scattering is isotropic. What is the intuitive physical reason for the isotropy in this case? Hint: examples of isotropic scattering can be found in Ch. 7 or 9.

33. Electromagnetic interactions of an atomic electron with the nucleus may be represented by a Feynman diagram corresponding to the exchange of a virtual photon between the electron and the nucleus. The weak-neutral-current interaction between the electron and the nucleus is represented by a similar diagram where the virtual photon is replaced by a virtual \(Z\) boson. Based on this fact, make a rough order-of-magnitude estimate of the energy shift (in eV) of the ground state of atomic hydrogen associated with the existence of the weak-neutral-current interaction. Note: the energy shift discussed in this problem is associated with the parity conserving part of the neutral-current weak interaction.

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