PHY 137A (D. Budker) Midterm Review Problems

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Note: These problems are designed to be significantly more difficult than what you'll actually see on the midterm!

1. We will consider a hydrogenic ion with a Carbon-12 core, so $Z = 6$ and $A = 12$.

   (a) Describe the kind of radiation I would need to use to completely ionize this ion in its ground state. Calculate the maximum wavelength of radiation (in nm) which could be used for this purpose and its energy per photon (in eV).

   (b) Find the energy (in eV), frequency (in Hz), and wavelength (in nm) of the analogue of the Paschen-α line for this hydrogenic ion.

   (c) Calculate the approximate Bohr radius of the electron before and after this transition.

   (d) What would be the change in frequency (in MHz) of this line due to the isotope effect if we had a Carbon-14 ion (with $A = 14$ and the same Z) instead of a Carbon-12 ion?

   (e) Suppose that I now apply various external fields and consider all relativistic (and other - excluding spin!) corrections such that I totally break *all* the degeneracies in the spectrum of the Carbon-12 Hydrogenic ion, but only slightly. That is, suppose that the correction to the energy of each state in the spectrum is different, but small. Then, if I use a very accurate spectrometer, I would find that the single Paschen-α line has actually split into many different lines. How many lines would I see?

   (f) Find all transitions that are visible.

2. (This is the last problem on my midterm review notes, which I left as an exercise). Consider the following state (we assume that $m + 1 \leq l$):

$$\Psi = \frac{1}{\sqrt{2}} \left(|lm\rangle + |lm+1\rangle \right). \tag{0.1}$$

   Compute $\langle L_z \rangle$, $\langle L_x \rangle$, and $\langle L_x^2 \rangle$ in this state.