# Homework # 4. Due: Friday, 10/06/2000

1. This is a problem in linear algebra. Suppose **T** is a diagonolizable matrix. This means that there exists a non-singular matrix **S** such that the matrix $\mathbf{T}'=\mathbf{S}\mathbf{T}\mathbf{S}^{-1}$ is diagonal. Show that each of the columns of the matrix $\mathbf{S}^{-1}$ is an eigenvector of **T**.
   Hint: here is one possible strategy for solving this problem (which is really quite simple). Let $\lambda_i$ and $\mathbf{V}_i$ be the eigenvalues and the corresponding eigenvectors of **T**. First show that if we construct the matrix $\mathbf{S}^{-1}=(\mathbf{V}_1\ \mathbf{V}_2\ldots\ \mathbf{V}_n)$, then $\mathbf{S}\mathbf{V}_i$ is a vector (column) that has zeros everywhere except for 1 in the i-th position. Next, just evaluate $\mathbf{S}\mathbf{T}\mathbf{S}^{-1}$ directly taking into account that $\mathbf{T}\mathbf{V}_i=\lambda_i\,\mathbf{V}_i$ and see what you get.

2. Consider a system of two atomic energy levels described by the Hamiltonian

$$\hat{H}=\begin{pmatrix} 0 & dE \\ dE & \varepsilon \end{pmatrix}.$$

   Here $\varepsilon$ represents the initial energy separation between the levels in the absence of the external electric field $E$, and $d$ is the *dipole moment*, a parameter which characterizes the strength of the system's coupling to the electric field.
   a) Find the energy levels and eigenstates of the system in the presence of the electric field.
   b) Analyze these results in two limiting cases: $|dE|\gg\varepsilon$ and $|dE|\ll\varepsilon$.
   c) Explain why electric field always shifts ground states of atoms towards lower energies.
   d) Sketch the dependence of the energy levels of the system on $\varepsilon$ for $|dE|=0$. Consider both positive and negative values of $\varepsilon$.
   e) Same for $|dE|\neq 0$.

3. Use the uncertainty principle to estimate a characteristic scale (in MeV) for the energy of nuclear excitations (i.e. energy difference between low-lying levels of a nucleus). Use the fact that the size of a nucleus is ~ 1-10 fm (1 fm = $10^{-13}$ cm; this unit is called a *femtometer*, or a *fermi* in honor of one of the giants of the 20-th century physics, Enrico Fermi), and that the mass of a *nucleon* (a proton or a neutron) is $\approx 2\cdot 10^{-24}$ g.