Homework # 3. Due: Friday, 09/22/2000

1. Consider so-called gravity waves on the surface of deep water (do not worry, we will get to quantum mechanics later on in this problem). You see these waves, for example, forming a spreading angle behind a moving boat whose vertex is at the front of the boat. It can be shown that frequency of such waves scales with the wavelengths as

$$\omega \propto \lambda^{-1/2}. \tag{1}$$

   a) Derive or at least explain this result. Hint: you can use, for example, the method of dimensions, and/or analogy with a pendulum. (If you figure out the proportionality constant in (1) this will be surely rewarded by extra credit points).

   b) Use Equation (1) to determine the ratio of the group and the phase velocity of such waves.

   c) Compare the ratio obtained in (b) with that for a free quantum-mechanical particle.

2. Consider a plane optical wave with finite temporal duration, i.e. a light pulse. As this pulse propagates in vacuum, its spatial and temporal extent remains unchanged. Compare this to the behavior of a quantum mechanical wave packet corresponding to a free particle with mass m. Hint: consider the dispersion (i.e. frequency dependence) of the group velocity for the two cases.

3. Griffiths' problem 2.26. Double δ-function potential: bound states. Hint: it can be shown that if the potential is symmetric, $V(x)=V(-x)$, then a bound steady-state solution of the Schrödinger equation should be either symmetric [$\psi(-x) = \psi(x)$], or antisymmetric [$\psi(-x) = -\psi(x)$]. Consider these two cases separately.

4. Find the reflection coefficient for scattering off the double δ-function potential from the previous problem. Assume that $E >> \frac{m\alpha^2}{2\hbar^2}$. Analyze your result as a function of separation between the wells. Comment on the analogy between this problem and interference in optics.