

# PROSPECTS FOR MEASURING PARITY NONCONSERVATION IN HYDROGENIC IONS USING HIGH-ENERGY ACCELERATORS

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Possibilities of precision weak-interaction measurements in relativistic hydrogenic ions are considered. It is shown that using high-energy ion storage rings (RHIC, SPS, LHC), and utilizing relativistic Doppler tuning and laser cooling, it is possible to achieve sensitivity necessary for testing physics beyond the Standard Model.

## 1 Introduction

Parity nonconservation (PNC) in atoms was first observed almost two decades ago.<sup>1,2</sup> These experiments established the existence of electron-nucleon neutral currents with structure and coupling as predicted by the Weinberg-Salam model. They were performed with heavy atoms, where PNC effects are greatly enhanced.<sup>3</sup> Since the first observations, a number of other experiments have been performed on heavy atoms,<sup>4</sup> and several experiments have reached the level of experimental uncertainty of 0.3-2% of the PNC effects. Atomic calculations necessary to interpret these experiments in terms of electroweak interaction parameters are currently at a comparable level of uncertainty. These experiments, combined with atomic calculations, provide a quantitative test of the Standard Model at small momentum transfer, complementary to the regime of high momentum transfer explored with high-energy particle accelerators.<sup>5</sup>

Further improving the sensitivity of atomic experiments by one-two orders of magnitude would put stringent limits on possible extensions to the Standard Model. However, the traditional experimental techniques may be approaching the limits of

their sensitivity. Moreover, the relativistic many-body calculations of the atomic PNC effects become much more difficult at the accuracy required. Therefore, it is very desirable to devise better approaches to atomic PNC. A promising direction is PNC experiments with rare earth atoms (see e.g. Refs. 6,7,8). Here one takes advantage of PNC-enhancement due to proximity of opposite parity states. In addition, uncertainties in atomic calculations can be avoided by comparing PNC effects on different isotopes. However, uncertainties in knowledge of neutron distributions within nuclei will start playing a significant role at the desired level of sensitivity.<sup>9</sup> Note also that isotopic PNC comparisons and single-isotope measurements probe different electroweak radiative corrections.<sup>5,4</sup> Other approaches to neutral current measurements at low momentum transfer, including neutrino scattering and polarized electron scattering are discussed in Ref. 10.

Here, we discuss a possibility to carry out PNC experiments in relativistic hydrogenic ions.<sup>11</sup> Due to their simple atomic structure, high precision theoretical calculations can be carried out in these ions. In addition, neutron distribution uncertainties will not present a serious problem in relatively light ions considered here, both because the structure of light nuclei is much better understood than that of the heavy nuclei, and because the electron wavefunction gradient at the nucleus is relatively small for light nuclei.

Various possibilities for PNC experiments with hydrogenic and helium-like ions have been discussed in the literature. Most of the early work dealt with the PNC contribution to the  $2S \rightarrow 1S + 1\gamma$  spontaneous decay<sup>12,13</sup> (an experimental upper limit on PNC-mixing in hydrogen-like argon was obtained in Ref. 14), while PNC in transitions between metastable states<sup>15,16,17,18</sup> and in Auger-electron emission<sup>19</sup> was discussed more recently. Unfortunately, none of the possibilities discussed in these papers seem to offer realistic ways to observe PNC effects, much less to achieve the high precision desirable today.

Here, we show that PNC experiments with ions become possible, in view of recent developments in relativistic ion colliders, high-brightness ion sources, laser cooling methods of ions in storage rings, etc. In particular, PNC experiments in relativistic hydrogenic ions involving laser-induced  $1S + 1\gamma \rightarrow 2S$  transitions can be carried out using the heavy ion accelerators RHIC, SPS and LHC. Relativistic Doppler tuning is used to tune laser light (photon energy  $\sim$  a few eV) in resonance with the ion transition ( $\sim$  keV in the ion rest frame) also allowing laser cooling, and polarization of ions by optical pumping. It is estimated that the PNC effect can be measured with statistical uncertainty  $\sim 10^{-3}$  in about a week of running time.

## 2 PNC in Hydrogenic Systems

In this paper we consider the conceptually simplest variant of a PNC experiment in a hydrogenic system: circular dichroism on the  $1S \rightarrow 2S$  transition (Fig. 1) in the absence of external electric and magnetic fields. Dichroism arises due to interference between the  $MI$  and the PNC-induced  $EI$  amplitudes of the transition. This approach allows one to estimate the required ion beam and laser parameters, and the achievable statistical sensitivity. In practice, it will be necessary to use one of the schemes involving ion polarization and external electric and magnetic fields.<sup>13,4</sup> These schemes do not change statistical sensitivity in an idealized experiment. In practice, however, they often allow to reduce the influence of technical noise. They also allow for reversals providing efficient control of systematic effects. A detailed discussion of possible interference schemes will be given elsewhere.

Due to the PNC interaction, the  $2S$  state acquires an admixture of the  $2P_{1/2}$  state:

$$|2S\rangle \Rightarrow |2S\rangle + i\eta|2P\rangle, \quad i\eta = \frac{\langle 2P|\hat{H}_w|2P\rangle}{E_{2S} - E_{2P}}. \quad (1)$$

Here  $\hat{H}_w$  is the weak interaction Hamiltonian,  $E_{2S,2P}$  are energies of the corresponding states, and the mixing coefficient  $i\eta$  is pure imaginary as a consequence of time-reversal invariance (we do not consider T-violating effects here). The magnitude of the PNC admixture is probed by tuning the laser in resonance with the highly forbidden  $1S \rightarrow 2S$   $MI$ -transition and observing circular dichroism, i.e., the difference in transition rates for right- and left-circularly polarized light. In order to consider PNC experiments with hydrogenic ions, it is convenient to trace  $Z$ -dependences of various atomic parameters. For convenience of reference, approximate formulae for these parameters including  $Z$ -dependences are collected in Table 1. A more detailed discussion can be found e.g. in Refs. 12,13,20.

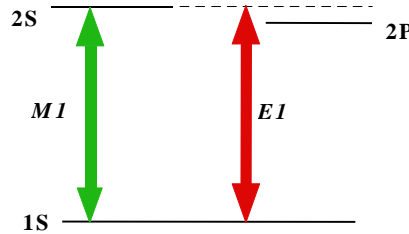


Fig. 1. The  $1S \rightarrow 2S$  transition in a hydrogenic system.

Table 1: Z-dependence of atomic characteristics for hydrogenic ions. In the given expressions,  $\alpha$  is the fine structure constant,  $\hbar=c=1$ ,  $m_e$  is the electron mass,  $G_F$  is the Fermi constant,  $\theta_w$  is the Weinberg angle, and  $A$  is the ion mass number.

Parameter	Symbol	Approximate Expression
Transition Energy	$\Delta E_{n-n'}$	$\frac{1}{2} \left( \frac{1}{n^2} - \frac{1}{n'^2} \right) \alpha^2 m_e \cdot Z^2$
Lamb Shift	$\Delta E_{2S-2P}$	$\frac{1}{6\pi} \alpha^5 m_e \cdot Z^4 \cdot F(Z)^a$
Weak Interaction Hamiltonian	$\hat{H}_w$	$i \sqrt{\frac{3}{2}} \cdot \frac{G_F m_e^3 \alpha^4}{64\pi} \cdot \left\{ (1 - 4 \sin^2 \theta_w) - \frac{(A-Z)}{Z} \right\} \cdot Z^5$
Electric Dipole Amplitude (2S $\rightarrow$ 2P $_{1/2}$ )	$EI_{2S\rightarrow 2P}$	$\sqrt{\frac{3}{\alpha}} \cdot m_e^{-1} \cdot Z^{-1}$
Electric Dipole Amplitude (1S $\rightarrow$ 2P $_{1/2}$ )	$EI$	$\frac{2^7}{3^5} \sqrt{\frac{2}{3\alpha}} \cdot m_e^{-1} \cdot Z^{-1}$
Forbidden Magn. Dipole Ampl. (1S $\rightarrow$ 2S)	$MI$	$\frac{2^{5/2} \alpha^{5/2}}{3^4} \cdot m_e^{-1} \cdot Z^2$
Radiative Width	$\Gamma_{2P}$	$\left( \frac{2}{3} \right)^8 \alpha^5 m_e \cdot Z^4$

<sup>a</sup> The function  $F(Z)$  is tabulated in Ref. 12. Some representative values are:  $F(1)=7.7$ ;  $F(5)=4.8$ ,  $F(10)=3.8$ ;  $F(40)=1.5$ .

### 3 Relativistic Doppler Tuning

Consider an ion with relativistic factor  $\gamma = 1/\sqrt{1-\beta^2} \gg 1$  colliding head-on with a photon of frequency  $\omega_{lab}$  in the laboratory frame. In the ion rest frame, the frequency is given by:

$$\omega_{ion\ frame} = \gamma(1 + \beta)\omega_{lab} \approx 2\gamma\omega_{lab} . \quad (2)$$

In order to tune to the  $1S \rightarrow 2S$  resonance for a hydrogenic ion, it is necessary to satisfy the condition:

$$\Delta E_{2S-2P} \approx Z^2 10.2 \text{ eV} = 2\gamma\hbar\omega_{lab} . \quad (3)$$

The relevant parameters of the storage rings RHIC,<sup>21</sup> SPS,<sup>22</sup> and LHC<sup>22</sup> are collected in Table 2. With visible and near-UV lasers, it is possible to access  $1S \rightarrow 2S$  transitions for ions with  $Z$  up to  $\approx 11$  (Na) at RHIC. For LHC, the corresponding  $Z$  range is up to  $Z \approx 48$  (Cd). In the following, we use RHIC parameters for numerical estimates.

Table 2. Parameters of relativistic ion storage rings.

Parameter	RHIC	SPS	LHC
$\gamma_{\max}$ for protons <sup>a</sup>	250	450	7000
Number of ions/ring <sup>b</sup>	$\sim 5 \cdot 10^{11}$	$\sim 2 \cdot 10^{11}$	$\sim 5 \cdot 10^{10}$
Number of bunches/ring	57	128	500-800
R.m.s bunch length	84 cm	13 cm	7.5 cm
Circumference	3.8 km	6.9 km	26.7 km
Energy spread w/o laser cooling	$2 \cdot 10^{-4}$	$4.5 \cdot 10^{-4}$	$2 \cdot 10^{-4}$
Normalized Emittance (N.E.)	$\approx 4 \pi \cdot \mu\text{m} \cdot \text{rad}$	$\approx 4 \pi \cdot \mu\text{m} \cdot \text{rad}$	$\approx 4 \pi \cdot \mu\text{m} \cdot \text{rad}$
Dipole field	3.5 T	1.5 T	8.4 T
Vacuum, cold	$< 10^{-11}$ Torr (H <sub>2</sub> , He)	-	$< 10^{-11}$ Torr (H <sub>2</sub> , He)

<sup>a</sup> For hydrogenic ions,  $\gamma_{\max}^{ions} = \gamma_{\max}^p \cdot Z - 1/A$

<sup>b</sup> Estimated from proton and heavy ion data.

#### 4 PNC Experiment: Estimate of Statistical Sensitivity and Laser Power; Processes leading to loss of ions

Consider circularly polarized laser light tuned in resonance with the  $1S \rightarrow 2S$  transition of an ion. The width of this transition is dominated by the Doppler width ( $\Gamma_D$ ) due to the ion energy spread:

$$\Gamma_D = (\omega \cdot \Delta\beta)_{ion\ frame} \approx \omega_{ion\ frame} \cdot \frac{\Delta\gamma}{\gamma}. \quad (4)$$

The fraction of ions excited from the ground state to 2S is given by:

$$\chi_{M1} = (M1 \cdot \tilde{B}\tau)^2 \cdot \frac{1}{\Gamma_D\tau}. \quad (5)$$

Here  $\tilde{B} = \tilde{E}$  is the laser field,  $\tau$  is the ion-laser interaction time. Here and below, we write all quantities in the ion bunch center of mass frame, unless specified otherwise. In this expression, the first term gives the transition probability for a group of ions in resonance with the laser field; the width of this group in the energy distribution is given by  $1/\tau$ . The second term gives the fraction ( $\ll 1$ ) of ions in the resonant group compared to the entire energy distribution. In the ongoing, we also assume that the laser spectral width is no greater than  $1/\tau$ .

From the point of view of sensitivity to PNC, it is desirable to have as many 1S $\rightarrow$ 2S events as possible; thus, one desires to use as high as possible laser power. However, there is an upper limit on laser power. In addition to the 1S $\rightarrow$ 2S transition, there is off-resonant excitation of the 1S $\rightarrow$ 2P $_{1/2}$  transition, which leads to optical pumping into the 1S Zeeman component decoupled from the laser light (for simplicity, we consider an ion with zero nuclear spin). The optical pumping saturation parameter (for  $\Gamma_D \ll \Delta E_{2S-2P}$ ) is given by:

$$\chi_{E1} = \frac{(E1 \cdot \tilde{E})^2}{4(\Delta E_{2S-2P})^2} \cdot \Gamma_{2P}\tau. \quad (6)$$

In order to avoid significant bleaching due to optical pumping, it is necessary to have  $\chi_{E1} \lesssim 1$ . Later we will see that one actually has to operate at somewhat lower saturation parameter in order to avoid ion loss due to photoionization. From (5) and (6) and expressions from Table 1, we have:

$$\chi_{M1} = \frac{3^8 \alpha^9 F^2(Z) Z^8}{2^{15} \pi^2 \frac{\Delta\gamma}{\gamma}} \chi_{E1}. \quad (7)$$

The total number of ions excited into the 2S state by light of a given circular polarization during the total time of experiment  $T$  (assuming the polarization is in each state half of the time) is  $N_{\pm} \approx \chi_{M1} \cdot \dot{N}_{ions} T / 2$ .

Here  $\dot{N}_{ions}$  is the average number of ions entering the interaction region per unit time. The PNC effect is given by the asymmetry:

$$P = \frac{N_+ - N_-}{N_+ + N_-} = \frac{2H_w}{\Delta E_{2S-2P}} \cdot \frac{E1}{M1}. \quad (8)$$

This corresponds to a statistical uncertainty:

$$\delta P = \frac{1}{\sqrt{N_+ + N_-}} = \frac{E1}{M1} \cdot \frac{\sqrt{\Gamma_D \Gamma_{2P} \chi_{E1}^{-1}}}{2\Delta E_{2S-2P}} \cdot \frac{1}{\sqrt{\dot{N}_{ions} T}}, \quad (9)$$

which yields:

$$\delta H_w = \frac{1}{4} \sqrt{\frac{\Gamma_D \Gamma_{2P}}{\dot{N}_{ions} T \chi_{E1}}}. \quad (10)$$

This shows that for an optimally designed PNC experiment, the statistical sensitivity is completely determined by the total number of available ions and by the transition widths. Combining Eqns. (10) and (4) and the expressions in Table 1, one finds that in order to obtain a certain sensitivity to  $\sin^2 \theta_w$ , it is necessary to have exposure  $\dot{N}T$  which is conveniently represented in units of particle-Amperes $\times$ year:

$$Exposure[part.Amp \times year] \geq \frac{\Delta\gamma}{\gamma} \cdot \frac{0.1}{Z^4 \cdot (\delta \sin^2 \theta_w)^2 \cdot \chi_{E1}}. \quad (11)$$

In order to proceed with the numerical estimate of the necessary exposure, we now briefly consider several processes specific to non-bare ions leading to reduction of ion lifetime in a storage ring, and discuss the effect of these processes on the proposed PNC experiments.

**Residual Gas Ionization.** For relativistic ions, the stripping cross-section in a collision with a residual gas atom can be estimated as (more detailed calculations can be found in Ref. 23):

$$\sigma = 4\pi\alpha^2 a_B^2 \frac{Z_a(Z_a + 1)}{Z^2}. \quad (12)$$

Here  $a_B$  is the Bohr radius,  $Z_a$  and  $Z$  are atomic numbers of the residual gas atom, and the ion, respectively. Estimates based on Eqn. (12) show that with residual gas pressure as given in Table 2, ion lifetime limited by this process is  $\approx$ half-hour for  $Z = 10$ .

**Field ionization.** The dipole magnetic field of a storage ring gives an electric field component  $\mathcal{E} \approx \gamma B_D$  in the ions' frame. This field (e.g.,  $\mathcal{E} \sim 10^9$  V/cm for RHIC), which is on the order of the atomic unit of electric field ( $\mathcal{E}_{at} = e/a_B^2 \approx 5 \cdot 10^9$  V/cm), can produce ionization of a non-naked ion. The lifetime with respect to field ionization (in the laboratory frame) for hydrogenic ions is:<sup>24</sup>

$$\tau_{f.i.}^{-1} = 4 \frac{\alpha c}{a_B} Z^5 \frac{\mathcal{E}_{at}}{B_D} \exp\left(-\frac{2\mathcal{E}_{at} Z^3}{3\gamma B_D}\right). \quad (13)$$

Substituting relevant storage ring parameters from Table 2 into Eqn. (13), one finds that field ionization is completely negligible in the cases of interest for the proposed experiments.

**Laser-induced photoionization.** Ions excited to the virtual  $2P_{1/2}$  state can absorb another photon from the laser beam and photoionize. The corresponding photoionization cross-sections are given in Ref. 25. For the photon energy corresponding to the  $1S \rightarrow 2S$  resonance, we have:

$$\sigma_{2P}^{ioniz} \approx 1.4 \cdot 10^{-2} \frac{a_B^2}{Z^2}. \quad (14)$$

Using this cross-section, Eqn. (6), and the formulae in Table 1, one estimates the photoionization probability in one laser-pulse interaction:

$$W \approx 1.2 \cdot 10^3 \frac{a_B}{z_R} \frac{\gamma \cdot F^2(Z)}{Z^2} \cdot \chi_{E1}^2. \quad (15)$$

Here we assume that the laser and the ion beam profiles are matched, and the Rayleigh range for the optical beam is  $z_R$ . (Since the ion beam emittance (see Table 2) is less than that of a laser beam:  $\epsilon_{tr} = (\text{N.E.})/\gamma < \lambda_{laser}/4\pi$ , the relevant Gaussian beam parameters are determined by the optical beam.) Using the RHIC parameters, and choosing  $z_R = 1$  m,  $Z = 10$ , and requiring the lifetime due to photoionization to be 1 hr, one finds the necessary value of the saturation parameter:  $\chi_{E1} \approx 6 \cdot 10^{-2}$ .

Note that photoionization losses due to population of the  $2S$  state rapidly scale with  $Z$  ( $\propto Z^{10}$  relative to photoionization from  $2P$ ), and may become important for  $Z \approx 40$ .



It may also become important in a Stark-PNC interference scheme if a sufficiently strong electric field is applied increasing the 1S→2S transition rate.

Let us now return to the estimate of the necessary exposure in a PNC experiment (Eqn. (11)). As an example, for  $\delta \sin^2 \theta_w = 10^{-3}$ , using Ne ions ( $Z = 10$ ) in RHIC, and substituting  $\chi_{E1} = 6 \cdot 10^{-2}$ , one obtains the necessary running time  $\sim 1$  week. In this estimate we assumed  $\Delta\gamma/\gamma = 10^{-6}$ , which is possible to achieve using laser cooling<sup>26</sup> as we discuss below.

Let us now estimate the laser power required for maintaining a given value of  $\chi_{E1}$  (see Eqn. (6)). We first assume that the laser operates in pulsed mode with repetition rate matched to the rate of arrival of ion bunches into the interaction region. The beam cross-section at the waist is  $2\pi\sigma^2 = \lambda_{laser} \cdot z_R / 2$ . Using the formulae in Table 1, one obtains an estimate for required number of photons in a laser pulse:

$$N_{phot} = \frac{\tilde{E}^2 \cdot 2\pi\sigma^2 \cdot 2\tau}{8\pi \cdot \Delta E_{1S-2P}} \approx 10^2 \cdot F^2(Z) \cdot \gamma^2 \frac{z_R}{\lambda_{laser}} \cdot \chi_{E1}. \quad (16)$$

For  $\gamma = 100$ ,  $Z = 10$ ,  $z_R = 1$  m and  $\chi_{E1} = 6 \cdot 10^{-2}$ , we have  $N_{phot} \approx 1.2 \cdot 10^{12}$ . The number of scattered laser photons is equal to the number of ions in a bunch  $\times \chi_{E1}$  ( $\approx 10^{10} \times \chi_{E1}$  for RHIC). This corresponds to light pulse energy  $\approx 1.2$   $\mu$ J, absorbed light energy  $\approx 0.6$  nJ, and the average absorbed light power  $\approx 3$  mW (57 bunches  $\times$  76 kHz). Laser parameters necessary for the proposed experiments are achievable with present technology.

## 5 Detection of Ions Excited into the 2S State

In hydrogen experiments, atoms in the 2S state are conveniently detected by applying electric field which mixes 2S and  $2P_{1/2}$ , and observing the resulting fluorescence. An alternative to the dc electric field is laser excitation to one of the higher-lying P-states. Unfortunately, it becomes increasingly difficult to use these techniques for the proposed experiment when the ion beam is ultrarelativistic. Indeed, essentially all fluorescence photons are emitted in the forward direction in a narrow angle  $\theta \sim 1/\gamma$ . It is difficult to discriminate these photons from the (orders of magnitude more numerous) laser photons back-scattered due to the 1S→2P resonance, since all photons arrive at a detector within a narrow time interval. It is also impossible to turn ions with a bending magnet by an angle  $\sim 1/\gamma$ , because ions

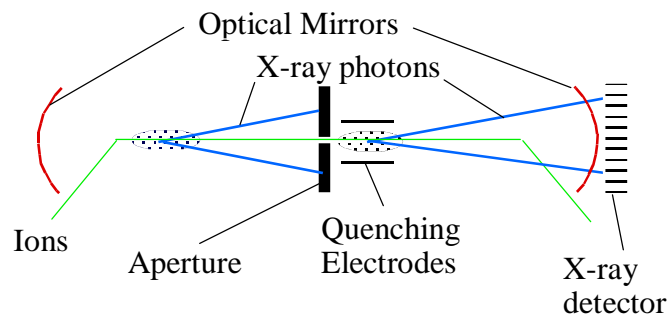


Fig. 2. A possible detection scheme for ions excited to the 2S state involving electric field quenching. This scheme can be realized for relatively low  $\gamma$  (e.g., for  $\gamma \approx 100$  at RHIC).

in the 2S state quench by the electric field arising in the ion's frame before the rotation is accomplished.

A detection scheme involving 2S quenching can be realized for relatively low  $\gamma$  (Fig. 2). For example, in the case of RHIC where  $\gamma \approx 100$  and the length of a straight section is 18 m, the experimental arrangement could be the following. The ion-laser interaction region  $\sim 1$  m long is arranged in the beginning of the straight section. About 10 m downstream from the interaction region, the ion beam passes through a collimator  $\sim 1$  cm in diameter. This collimator serves to absorb the back-scattered laser photons traveling along the beam axis in an angle greater than  $\sim 10^{-3} \ll 1/\gamma$ . The 2S-quenching region is located after the collimator; a position-sensitive X-ray detector located several meters from the quenching region detects photons resulting from quenching with nearly 100% efficiency (the remaining back-scattered photons in the narrow angle  $\sim 10^{-3}$  can be used for alignment and diagnostics).

Another commonly used technique for detection of ions in metastable states is photoionization. In this case, however, photoionization will lead to loss of ions from the storage ring, leading to reduced beam lifetime and loss of statistical sensitivity.

A possible way to detect ions in the 2S state is by measuring absorption of a laser beam tuned to a  $2S \rightarrow nP$  transition ( $n \geq 3$ ). Detection of small absorption can be carried out by arranging a multi-pass system, or a resonant cavity for the laser beam. The necessary number of passes can be estimated from the condition that the total absorption is  $\sim 1$ . This straightforward calculation shows that if one uses an optical cavity with number of passes  $\sim 10^4$ , in order to avoid loss in statistical

sensitivity, it will be necessary to apply a Stark-field, so the Stark-induced  $1S \rightarrow 2S$  transition amplitude significantly exceeds the  $MI$  amplitude.

## 6 Systematic Effects

A detailed discussion of systematic effects is specific to a concrete scheme of a PNC experiment and thus lies beyond the scope of this paper. Here we limit ourselves to a brief discussion of several general points relevant to experiments with hydrogenic ions.

First of all, it should be noted that the same  $2S-2P_{1/2}$  near-degeneracy that leads to enhanced PNC mixing between these states, also leads to enhanced mixing due to electric fields. It was just this high sensitivity to stray fields that caused systematic effects that ultimately limited the sensitivity of PNC experiments with hydrogen (for a review of hydrogen experiments, see e.g. Ref. 27).

As a measure of sensitivity to stray fields, it is useful to see which electric field  $\mathcal{E}_{st}$  (in the ions' frame) causes  $2S-2P_{1/2}$  mixing of the same magnitude as the PNC interaction. From the formulae in Table 1, it follows that  $\mathcal{E}_{st} \approx 2 \cdot 10^{-9} \cdot Z^6$  V/cm. Therefore, the relative sensitivity to stray fields rapidly falls with  $Z$ . It should be emphasized that in Stark-PNC interference experiments, the PNC signature can only be mimicked by a product of two or more imperfections (such as stray fields, misalignments, non-reversals, etc.); therefore, control of stray fields is actually necessary only at levels several orders of magnitude larger than the values given above.

A characteristic value of electric field produced by the ions' space charge (in the ions' rest frame) can be estimated as:

$$\mathcal{E}_{s.c.} = \frac{2\rho}{b} = \frac{2N_{ions}(Z-1)e}{\gamma \cdot l \cdot b}. \quad (17)$$

where  $\rho$  is the line density of charge,  $N_{ions}$  is the number of ions per bunch,  $l$  is the bunch length in the laboratory frame, and  $b$  is a characteristic transverse dimension of the beam. Using  $b = \sqrt{\epsilon_{tr} \beta^*} \approx 2 \cdot 10^{-2}$  cm (where  $\beta^* \approx z_R$  is the beta-function in interaction region), one obtains  $\mathcal{E}_{s.c.} \approx 100$  V/cm for RHIC. The Stark-induced  $1S \rightarrow 2S$  amplitude due to this field is much smaller than the  $MI$  amplitude; therefore, it does not reduce sensitivity of the PNC experiment. Of course, the average value of the space charge electric field seen by ions in a bunch is zero.

## 8 Laser Cooling and Intra-Beam Scattering

In the estimates above we have assumed  $\Delta\gamma/\gamma=10^{-6}$ . This approximately corresponds to the Doppler limit for RHIC (assuming that the 1S $\rightarrow$ 2P transition is used for cooling):

$$\left(\frac{\Delta\gamma}{\gamma}\right)_D = \sqrt{\frac{\Gamma_{2P}}{M_{ion}}} \approx 4 \cdot 10^{-7} \text{ for } Z = 10. \quad (18)$$

In principle, laser cooling rates can be very fast.<sup>26</sup> However, similar to the situation discussed above, the laser power has to be low enough to avoid excessive ion losses due to photoionization from the 2P state. On the other hand, the cooling rate has to be high enough to compensate the heating effect of the intra-beam scattering. The cooling rate per turn can be estimated as:

$$\frac{1}{(\Delta\gamma/\gamma)^2} \frac{d(\Delta\gamma/\gamma)^2}{dn} = -2\chi_{cooling} \cdot \frac{\hbar\omega_{laser}}{Mc^2} \cdot \frac{\Gamma_{2P}z_R}{c} \cdot \frac{\gamma}{\Delta\gamma}. \quad (19)$$

where

$$\chi_{cooling} = \frac{(E1 \cdot \tilde{E})^2}{\Gamma_{2P}^2}. \quad (20)$$

The heating rate due to intra-beam scattering can be estimated as (a detailed discussion of intra-beam scattering can be found e.g. in Ref. 28):

$$\frac{1}{(\Delta\gamma/\gamma)^2} \frac{d(\Delta\gamma/\gamma)^2}{dn} \approx \frac{N_{ions} \cdot r_{ion}^2 \cdot C}{\gamma^3 \epsilon_{tr}^{3/2} \sqrt{\beta^*} \cdot l} \left(\frac{\gamma}{\Delta\gamma}\right)^2. \quad (21)$$

where  $r_{ion} = Z^2 e^2 / Mc^2$  is the classical radius of an ion,  $C$  is the storage ring circumference. For RHIC and  $\Delta\gamma/\gamma = 10^{-6}$ , from Eqn. (21), one estimates that without cooling  $(\Delta\gamma/\gamma)^2$  would double in about 3,000 turns. In steady state conditions, the cooling rate (19) should be equal to the heating rate (21). This gives the minimum required value of the cooling saturation parameter  $\chi_{cooling} \approx 10^{-4}$ . This corresponds to a significantly lower laser power than for the PNC measurement. Ionization losses due to the cooling process can be estimated as above; they correspond to ion lifetime  $\approx 30$  hrs. Therefore, it appears possible to achieve the necessary energy spread.

## 9 Conclusions

We have shown that it is feasible to perform sensitive parity-violation measurements in relativistic ions. These experiments become possible due to modern developments in relativistic ion storage rings, lasers, and laser cooling. Of particular importance for these experiments are high ion currents, long ion lifetime in a storage ring, small emittance and energy spread. High ion energies make it possible to tune visible and near-UV lasers in resonance with transitions of interest. The proposed technique has sensitivity sufficient for testing physics beyond the Standard Model. Further development in ion injectors will lead to higher stored currents which may allow even more sensitive PNC experiments in the future.

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$$\hat{H} = -\frac{Ze^2}{r} + \frac{\hat{p}^2}{2m_e} - e\vec{\epsilon} \cdot \vec{r}$$

for an ion is reduced to the hydrogen Hamiltonian by substituting

$$e = \frac{e'}{\sqrt{Z}}, \epsilon = \epsilon' \sqrt{Z} .$$

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