ATOMIC PHYSICS

An Exploration Through Problems and Solutions

Dmitry Budker
Department of Physics, University of California, Berkeley, USA

Derek F. Kimball
Department of Physics, University of California, Berkeley, USA

and

David P. DeMille
Physics Department, Yale University, USA

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To our teachers

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CONTENTS

PREFACE

We have found that usually the best way to learn something new is to ask concrete questions and try to work out the answers. Often some of the simplest questions have surprising and unexpected answers, and some seemingly complex problems can be solved in a simple way. In this book we have collected some of these problems and our solutions to them. The book encompasses many issues we faced as we ourselves made the transition from undergraduate students to practicing experimental atomic physicists and instructors. However, the text is not intended to be comprehensive, but rather addresses various aspects of atomic physics which we have found interesting and important.

In the course of doing atomic physics, we always find ourselves crossing boundaries into other subfields; the selection of problems reflects this gray area. It also reflects our specific interests, with several problems about symmetry violation, etc. that would not appear in more "standard" textbooks. It is our philosophy that working on specific problems usually helps with understanding of more general issues, and indeed may be the most useful way to *really* learn anything. It is our hope that some selection of the broad range of problems given here will pique the interest of any reader, and thus initiate this process.

Where possible, we try to emphasize approximation methods, dimensional considerations, limiting cases, and symmetry arguments as opposed to formal mathematics. We often appeal to pictures, tables, and graphs. This problem-solving approach is aimed at developing intuition about physical principles and fosters the important ability to perform "back-of-the-envelope" calculations. These are the tools we find most useful when trying to solve the types of problems we commonly encounter in the laboratory. Of course, on occasion a formal mathematical approach (as painful as it could be) can lead to important insights. Generally, in order to deeply understand various aspects of physics, it is good to have both an intuitive picture as well as the appropriate mathematical tools.

This book is intended for advanced undergraduates and beginning graduate students interested in atomic, molecular and optical physics, and we assume that readers possess basic knowledge of quantum mechanics [at the level of Griffiths (1995), Bransden and Joachain (1989), or similar texts], electrodynamics [at the level of Griffiths (1999), Purcell (1985), or similar texts], and thermodynamics [at the level of Reif (1965), Kittel and Kroemer (1980), or similar texts]. However, we hope that many of the problems will also be of interest to professional scientists.

In physics, there continues to be a raging debate over what is the best system of units to use and whether or not units should be standardized. We feel that the choice of units is a personal one, especially since converting between different systems is relatively straightforward. That said, in this book we have a tendency to use CGS units, since we find them most convenient (especially in problems involving electromagnetism). We also set $\hbar = 1$ when it is convenient to do so and measure energies in frequency units, as is common practice in atomic physics (since energy measurements are typically performed by measuring frequencies).

Each problem in the book is intended to stand on its own. If there is a particular subject in atomic physics that one is interested in learning about, there may be a problem about it in this book. We envision the reader turning right to that page and starting to try to figure it out. Hopefully, at the end of this exercise, one will have gained some familiarity with the topic, enabling her or him to understand more advanced, specialized literature on the subject, or go straight to the lab and get to work!

In the introduction to most problems there is a brief discussion of the relevance of the topic to modern atomic physics with references to research literature on the subject. The cited references are not intended to be comprehensive, but merely provide a starting point in a search for more information about the subject of the problem. We apologize in advance to the innumerable scientists whose important contributions are not mentioned. Also, for a few problems, especially in subfields of atomic physics dear to our hearts, there are some historical remarks. Of course, there is a great deal of history surrounding almost all of the topics covered in this book, and we could not tell all of it. Nonetheless, we thought a few, not widely known stories might be enjoyable.

Some of the problems are written as tutorials on various subjects in atomic physics [they are marked with a (T)]. In such problems, there are a series of short questions that are intended to guide the reader through some important material. Hopefully the reader will find this more entertaining and interactive than just reading the explanation straight through.

We hope you enjoy reading and using the book as much as we have enjoyed writing it!

D. B.
D. F. K.
D. P. D.

Berkeley, California
May 2003

NOTATION

The following is a table of symbols commonly used throughout the book, their meaning, and their value where appropriate. In most locations, we remind the reader of the meaning of the symbols when they appear. Also see Appendix A for practical units, conversion factors, and typical values of various parameters.

Symbol	Meaning	Value
m, m_e	electron mass	9.1085×10^{-28} g
		$0.511\ \mathrm{MeV/c^2}$
m_p	proton mass	1.6726×10^{-24} g
		$938.28\ \mathrm{MeV/c^2}$
m_n	neutron mass	1.6750×10^{-24} g
		$939.57\ \mathrm{MeV/c^2}$
$m_n - m_p$	difference between nucleon masses	$1.293\ \mathrm{MeV/c^2}$
e	electron charge magnitude	4.8029×10^{-10} esu
h	Planck's constant	$6.6252 \times 10^{-27}\ \mathrm{erg \cdot s}$
$\hbar = h/(2\pi)$		$1.0544 \times 10^{-27}\ \mathrm{erg \cdot s}$
$\alpha = e^2/(\hbar c)$	fine structure constant	$1/137.036$
$a_0 = \hbar^2/(me^2)$	Bohr radius	5.292×10^{-9} cm
$\mu_0 = e\hbar/(2mc)$	Bohr magneton	$0.93 \times 10^{-20}\ \mathrm{erg/G}$
		$1.40\ \mathrm{MHz/G}$
$\mu_N = e\hbar/(2m_pc)$	nuclear magneton	$5.06 \times 10^{-24}\ \mathrm{erg/G}$
		$762\ \mathrm{Hz/G}$
$R_\infty = me^4/(4\pi\hbar^3 c)$	Rydberg constant	$109,737\ \mathrm{cm}^{-1}$
k_B	Boltzmann's constant	$1.38066 \times 10^{-16}\ \mathrm{erg/K}$
		$8.61735 \times 10^{-5}\ \mathrm{eV/K}$
L, l	orbital angular momentum	units of $\hbar$
	(total, individual particle)	
S, s	electron spin	units of $\hbar$
J, j	total electronic angular momentum	units of $\hbar$
I	nuclear spin	units of $\hbar$
F	total atomic angular momentum	units of $\hbar$

When we deal with spin-1/2 systems, we will commonly employ the notation $|+\rangle$ and $|-\rangle$ to denote the spin up ($m = +1/2$) and spin down ($m = -1/2$) states, respectively. Here m is the projection of the spin along the quantization axis.

The ubiquitous Clebsch-Gordan coefficients[1] describe the connection between the coupled basis $|J, M\rangle$ and the uncoupled basis $|J_1, M_1\rangle|J_2, M_2\rangle$ (where J, J_1, J_2 are angular momenta and M, M_1, M_2 are the projections of the respective angular momenta on the quantization axis):

$$|J, M\rangle = \sum_{M_1, M_2} C(J_1, J_2, J; M_1, M_2, M)|J_1, M_1\rangle|J_2, M_2\rangle \tag{2.1}$$

$$|J_1, M_1\rangle|J_2, M_2\rangle = \sum_{J,M} C(J_1, J_2, J; M_1, M_2, M)|J, M\rangle \ . \tag{2.2}$$

In the text we consistently use the notation:

$$C(J_1, J_2, J; M_1, M_2, M) \equiv \langle J_1, M_1, J_2, M_2|J, M\rangle \ , \tag{2.3}$$

and employ the commonly used phase convention of Condon and Shortley (1970), Edmonds (1996), and Sobelman (1992).

[1] The Clebsch-Gordan coefficients are also referred to as *vector-coupling coefficients*, *vector-addition coefficients*, and *Wigner coefficients* in the literature.