# Physics H7A (Budker) Fall 2009
# Midterm 1 Review
# October 6-7, 2009

**1. Lagrange points and pictures** Consider two massive objects, say the Earth and the Moon. Let the distance between them be $R$. If these objects were at fixed locations (i.e. not rotating about each other), there would be one location where a small satellite of mass $m$ could remain fixed. Draw a picture of the Earth-Moon system and label approximately where this points is.

Now consider the case where the Moon is orbiting the Earth. (So long as the Earth is significantly heavier than the Moon, which it is, we can assume the Earth is fixed and the Moon rotates about the Earth's center.) There are now *five* points in the plane of rotation where an object can remain relatively at rest in the rotating frame of the Moon about the Earth. These are known as the Lagrangian points. Find (very) roughly where they are located and explain, using a free body diagram, why the mass $m$ would be stable there.

**2. Multi-stage or single-stage rocket** A rocket of (empty) mass $m_0$ is designed with two identical boosters, which when ignited burn their fuel at a constant rate of $b = dm/dt$ for a time $T$. (So the initial mass of the rocket+fuel is $m_0 + 2bT$.) In the absence of gravity, is it better for the rocket to ignite its boosters sequentially or simultaneously? What about if the boosters have an (empty) mass $m_b$ and we're going to jettison the boosters once each is empty? What about in the presence of gravity?

**3. The tipping flagpole** A flagpole of linear mass density $\lambda$ (and length $\ell$) is planted in the ground at a slight angle $\theta_0$ from the vertical. In the absence of wind, this will cause the flagpole to gradually bend as one moves up the length. Consider a part of the flagpole a distance $s$ along its length.

a) Draw the FBD for a small bit of the flagpole of length $ds$. (Hint: The angle on the lower side will be $\theta$ and on the higher side will be $\theta + d\theta$. The tensions will be $T(s)$ and $T(s + ds)$, respectively.) Assume that tension only acts along the tangent line, like a rope.

b) Using one of the equations from the FBD, show that $T(s)\sin\theta$ is a constant along the length.

c) If you're feeling ambitious, try to solve for how the angle varies along the rope. You may find it easier to work with $x$ or $y$ as describing the point on the rope than using $s$.

**4. A MIRVed projectile** A projectile of mass $2m$ is fired with an initial velocity $v_0$ at an angle of $\theta_0$ to the horizontal. It has a timer so that at some point along its trajectory, it will explode horizontally into two bits, each of mass $m$. What should the timer be set to so that the total range of the forward-going projectile is the biggest? Does anything change if the projectile explodes not horizontally but along the tangent vector to its path?

**5. Goliath's doom** A rope has linear mass density $\lambda$ and length $\ell$ with a ball of mass $m$ at the end. The rope is swung at an angular velocity $\omega$. Find the tension at an arbitrary point $r$ along the length of the rope. Neglect gravity.

**6. The sliding dumbbell** Here is a modification of a classic problem in mechanics. A dumbbell with two point masses $m$ and length $\ell$ is laid vertically against a wall. The bottom mass of nudged so that it begins to slide. We want to find the angle where the top mass comes away from the wall. Do this problem in the following way.

a) First suppose that the masses are constrained to move along the $x$ and $y$ axes. Let the angle $\theta$ be the angle between the dumbbell and the wall. (It is initially zero.) Write the FBD for the two masses and find the equation of motion for $\theta$. You should find $\ddot{\theta} = \frac{g}{\ell}\sin\theta$ after a fair amount of work.

b) Using conservation of energy, show that $\omega^2 = \frac{2g}{\ell}(1 - \cos\theta)$ where $\omega = d\theta/dt$.

c) In the real problem, the top mass comes away from the wall when the horizontal constraint force becomes negative. (Since a normal force can only push, not pull.) Find the angle when this occurs. (You should find $\cos\theta = 2/3$.)