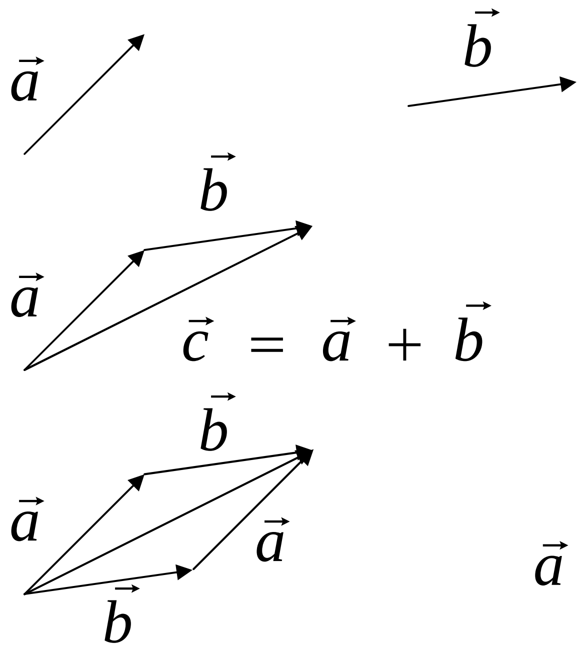
Plan of Lectures

Lecture one, August 27

- Introduction, names of Instructor and GSI, contact information, web page, class e-mail list, office hours, labs, discussion sections, midterm.
- The goal of the course – professional introduction to mechanics as the basis of all further physics
    - The role of math – just a tool; the ideas will be physical, not mathematical: dimensions, limiting cases, orders of magnitude vs. formal mathematical reasoning, example with a student, her boyfriend and ice-cream (odd number vs. too expensive).
    - Systems of units defined basically by units of length, mass, and time. Two most commonly used systems are SI (MKS) and Gaussian (CGS). Many working physicists still use CGS as it is particular convenient for E&M. In mechanics, it really does not matter, and we will use all kinds of units. Warning: watch out for unit consistency.
- Use of the K&K book. We will heavily rely on the book. I find it silly to repeat everything that is so well written there in class. So, in general, we will be going over examples that are not necessarily in the book, while I will be assuming that you have read the chapters that I will assign. For example, this week, please read Chapter One on mathematical preliminaries. Homework will include a mixture of "original" problems and those from K&K. It is quite essential to do at least half-a-dozen problems a week in order to keep up with the course.

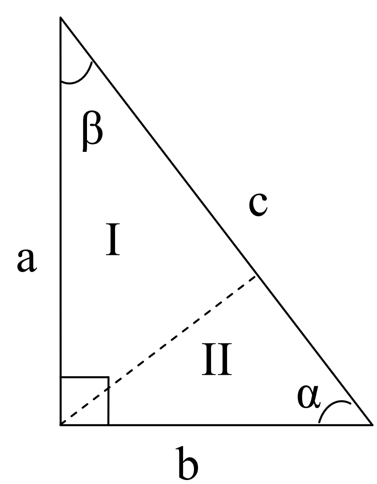
- Enough of preliminaries; time to get to business. Scalars vs. Vectors. Scalars are quantities that do not have any spatial direction associated with them (time, mass, number of items, distance), while vectors are directed line segments, with which we associate both a direction (generally, in 3-dimensional Cartesian space, but sometimes restricted to lower dimensions, e.g., 2 in the case of a plane, or 1 for linear problems), and a scalar representing the length of the vector. There are many notations people use for vectors, for example bold letters like **V**, and letters with arrows on top of them like $\vec{v}$. In the special case of vectors with unit length, a common notation is a "hat" on top of the letter: $\hat{a}$.
- Properties of vectors. Vectors can be added subtracted, multiplied or divided by scalar, you can use parenthesis, permute things pretty much in the usual way.
  An example: addition of two vectors:



$$\vec{a}+\vec{b}=\vec{b}+\vec{a}$$

Now you can also multiply two vectors, and there are two very different way of doing it:

- <u>Scalar Product:</u> $\vec{a}\cdot\vec{b}=|\vec{a}||\vec{b}|\cos\theta$, which is a scalar, and where vertical bars designate the length of the vector, and θ is the angle between the vectors; and <u>Vector Product</u>: $\vec{a}\times\vec{b}$ which is a vector that is directed according to the right-hand rule, and whose length is $|\vec{a}||\vec{b}|\sin\theta$. We will discuss this in some detail. Incidentally, there is generally another thing one can make out of two vectors called a <u>tensor</u>, which is neither a scalar nor a vector, but we will not deal with tensors for now.
- <u>Cartesian coordinates:</u> $\vec{a}=a_x\hat{x}+a_y\hat{y}+a_z\hat{z}$. Scalar and vector product expressed in the coordinate notation.



- We have already talked a bit about the difference in the way mathematicians and physicists think and about the importance of dimensions. Turns out that you can often say a lot about a problem just from knowing the units in which the answer should be measured. Here is an example (due to a great theoretical physicist A.B. Migdal), which is frowned upon by formal mathematicians. We will prove the Pythagoras' Theorem by the method of dimensions. The Theorem states that in a right-angle triangle the sum of the squares of the side lengths is equal to the square of the length of the hypotenuse: $a^2 + b^2 = c^2$.

To prove this, let's drop a perpendicular from the right-angle corner onto the hypotenuse as shown. Clearly, the area of the original triangle is the sum of the areas of triangles I and II. Now, we know that a right-angle triangle is fully defined by two of its elements, for example the length of the hypotenuse and one of the angles, say β. Since area is measured in m^2, i.e., it has dimensions of length square, we can write for the area of the whole triangle: $A=c^2\cdot f(\beta)$, where $f(\beta)$ is some dimensionless function that only depends on the angle β. Since, similarly, we have analogous expressions for the areas of triangles I and II, we immediately write $c^2\cdot f(\beta) = a^2\cdot f(\beta)+ b^2\cdot f(\beta)$, which yields the sought-for result upon canceling the common factor $f(\beta)$.

- Let us now turn to something else. Actually, this is an important thing because all modern science started some 400 years ago with this experiment – Galileo dropping objects from the leaning tower of Pisa (photo tour)

Lecture two, Sept. 1

More on Galileo's La Torre di Pisa experiments.

- Estimate the height of the tower (56 m)
- Estimate how long it takes a ball to fall (a few seconds)
- Use this example to introduce position vector, velocity vector, speed, average velocity, instantaneous velocity, acceleration
- Relation between position, velocity, acceleration
- Integrals and derivatives
- Galileo's hypothesis that balls fall the same, independent of mass, and general opposition to it; Aristotle; Philosophy vs. Natural Science
- Derivation and discussion of
    - $h = h_0 - gt^2/2$; limiting cases
    - $v = gt$
    - $t_{fall} = (2h/g)^{1/2}$; how does the fall time scale with the height?

- A bit more ballistics. Cannon shoots a cannonball at a certain initial speed v_0.
    - How far does the ball travel in the horizontal direction till it hits the ground?
    - At which angle should one tilt the barrel, so the cannonball flies the farthest?

  Solve in a couple of different ways; introduce optimization using the zero-derivative method. We can use this problem to once again highlight some general principles of how a mechanics problem may be solved:
    - Choose a convenient coordinate frame; if convenient, write separate eqns for independent motion along different coordinates
    - Are there any limiting cases for which the answer is obvious?
    - After the answer is obtained, does it have correct dimensions? Does it make sense in the limiting cases thought of above?
- The falling stone/feather demonstration (in air and in vacuum)
- Free fall/photogate demo

Lecture three, September 3

- The next topic I'd like to discuss is rotational motion. This is usually discussed somewhat later in the course; however, I'd like to introduce it now in order to illustrate an important concept: if you know one thing well, you automatically know a whole bunch of other things well. We will now see direct analogies between linear and circular motion. Other examples are, just to give you an idea, if you know how a pendulum works really well, you also know RLC circuits in electronics, waves in the ocean, oscillations in plasma, the structure of a light beam, etc, etc.
  - Uniform circular motion. Linear and angular velocity
  - Angle is the analog of linear coordinate, ω is analog of *v*
  - Derivation of v=ωR; refine to the vector form: $\vec{v} = \vec{\omega} \times \vec{R}$
  - Note that velocity for uniform motion, while of constant magnitude, is continuously changing direction → acceleration; derive $\vec{a} = \vec{\omega} \times \vec{v}$ so that ($a=v^2/R$)
  - Polar coordinates and vector representation of Θ and ω
  - What are the linear velocities of various points on a rolling wheel?
- Demo: trajectory of a point on a wheel
- If a train is moving from Moscow to St. Petersburg, are there any parts that are moving from St. Petersburg to Moscow?
- An aside: why the railroad gauge is different in the Eastern and Western Europe
- The 3+1 Newton's Laws
  - Difference in the way physical theory is built cf. mathematical axiomatics
  - The First Law; inertial frames
  - The Second Law $\vec{F} = m\vec{a}$; what is mass, force; $\vec{F}$ is the vector sum of all forces acting on the particle
  - The Third Law $\vec{F}_a = -\vec{F}_b$
  - The Universal Gravity Law $\vec{F}_{grav} = -G\frac{m_1 m_2}{r^2}\hat{r}$, where $G \approx 6.67 \cdot 10^{-11}$ Nm^2/kg^2; remarkably, the masses entering this law are the same as in the Second Law (the Equivalence Principle). The origins of gravity.

Lecture four, September 8

- Demos:
  - Water cannon
  - Falling balls; independence of the vertical motion from horizontal motion
  - Shoot the monkey
- Gravitational forces due to spherical objects. Derivation of the fact that a body of spherical shape exerts gravitational force on an external mass as if all its mass was concentrated in the center. Absence of the gravitational force within a spherical shell; Newton's argument. Observation that problems with simple answers usually have simple solutions.

- Alternative derivation (via the Gauss' Law) and various consequences of the above fact. Connection to electricity and magnetism.
- A problem on the application of the second Newton's law: monkey on a rope

Lecture five, September 10

- Challenge problems:
  - What is the optimal shape of a blob of play-dough to maximize gravitational pull at a point?
  - Cowboy with a lasso on a conical ice mountain
- More on monkey on a rope: the recipe for survival
- Subtleties of inertial vs. non-inertial frames. We defined an inertial frame as such a frame where a body does not accelerate in the absence of forces. Also, in an inertial frame, we have the Second Newton's Law. Does a frame which is free falling in the Earth's gravitational field qualify as an inertial frame? We can safely say that it does not because the bodies do not accelerate in this frame upon the action of the Earth's gravitation force, so the Second Law does not hold. However (and this is the tricky part), as a consequence of the equivalence principal, an observer in such a system cannot tell whether they are in an inertial frame, or a frame free-falling in the gravitational field (unless they see the Earth). So from the perspective of such an observer who is ignorant of the fact that there is a body (Earth) exerting gravitational pool, this would seem like a perfectly fine inertial frame…
- Rotating conical pendulum; stability analysis
- Demonstration of the conical pendulum – how a stable-equilibrium point becomes unstable
- Demonstration of an inverted pendulum
- The Coriolis force (arising when a body is involved in rotational motion and is changing the radius at the same time) – a straightforward derivation

Lecture six, September 15

- Demonstrations: gravitational attraction between lead balls (unsuccessful), First Newton's Law: sliding tablecloth, and pulling card from under weight
- Calculation of the Coriolis acceleration for a car moving from SF to LA with v=72 km/hr(=20 m/s)
- How do we decide whether a quantity is large or small? Example: the effect of Coriolis forces on weather systems (a calculation of forces on air resulting in wind; air density)
- A brief review of the Avogadro's law, and how one might go about calculating the density of air; atmospheric pressure in various units
- Momentum; generalization of the Second Law. Forces acting on composite systems – external and internal
- Momentum conservation; some simple examples
- Introduction to rocket Science – the Tsialkovskii formula

Lecture seven, September 17

- Demonstrations: water in rotating bucket, candles on rotating platform
- More rocket Science – the Tsialkovskii formula; estimate of the launch weight to payload weight ratio for space travel (including the estimate of the orbital speed for a low circular orbit around the Earth)
- Does it help much to launch from the equator?
- Demonstration of a compressed air rocket
- An example of momentum conservation: a fisherman in a boat problem; center of mass
- A more subtle case: water friction $\propto$ (-v)

Lecture eight, September 22

- Lots of demos: rockets, inertia (with bed of nails), …
- Energy, power; relation to Newton's laws
- Definition of work and power, units; derivation of $P = \vec{F} \cdot \vec{v}$
- Example: work and energy conversion when we lift a weight in gravitational field, and then drop it
- Energy conservation

Lecture nine, September 24

- More on fisherman and boat for the case of friction force $\propto$ (-v)
- The brachistochrone problem: demo; the brachistochrone property of the cycloid
- How to minimize in the space of functions? The Euler-Lagrange method
- Mathematical asides:
  - Partial derivatives
  - Taylor-series expansion

Lecture ten, September 29

- Experiment with rifle shooting into wooden block; determination of the muzzle speed of the bullet
- A brief discussion of dry friction
- Experiments verifying the "law" of friction: $(F_{fr})_{max}=\mu N$, where N is the magnitude of the normal force
- Experiment showing large difference between static and dynamic friction
- Demo: wooden plank oscillating on counter-rotating bicycle wheels
- Demo: block and tackle hoist
- Springs; the Hooke's law; parallel and series connections of springs (with demos); analogy with electrical capacitors

Lecture eleven, October 1

- How does a car's stopping time depend on its mass?
- Elastic collisions, demos with balls: the Newton's cradle; two balls of unequal mass. We notice and explain why the initially stationary ball comes to rest after every other collision (independently of the mass ratio!)
- Potential energy of a spring
- Demonstrations of the Hooke's law and harmonic oscillation. Measuring the dependence (or lack thereof) of the frequency on the mass and amplitude
- Derivation of the Simple Harmonic Oscillator (SHO) motion
- Energy transformation in a SHO
- Some other examples of SHO: pendulum, electrical LC circuit

Lecture twelve, October 6

- The falling-chain demo; calculation of the force on the scale
- Other demos: the Lissajous figures; turntable and pendulum
- Experiment in which we measure μ using the inclined surface method
- What is the maximum acceleration of a car (either positive or negative) in g's ?
- Oscillation of two masses connected with a spring; reduced mass
- Other examples of the use of reduced mass: planets, atoms, molecules
- Oscillations near minimum of a general potential: Taylor expansion to obtain SHO approximation

October 8: Midterm (need blue books)

Lecture thirteen, October 13

- Rotational dynamics: moment of inertia; moments of inertia of some simple configurations
- More examples of moments of inertia
- A brief discussion of the concept of the moment-of-inertia tensor and its principle axes
- Loop-de-loop demo and discussion
- Angular momentum and is conservation; demonstrations with rotating chair and bicycle wheel
- Precession
- Torque; $\dot{\vec{L}} = \vec{\tau}$ as analog of $\dot{\vec{p}} = \vec{F}$
- The problem of a disc on an inclined plane with friction; effective inertial mass; discussion of energy balance

Lecture fourteen, October 15

- The parallel axis theorem
- Planetary motion: the three Kepler's Laws and how they relate to angular momentum

- Gyroscopes: how can we understand precession from $\dot{\vec{L}} = \vec{\tau}$; a demo
- Derivation of the gyroscope's precession frequency
- Gyro in a suitcase, gyro-boat, and other fun demos
- Why do we say that the force of gravity is applied to the c.m.?

Lecture fifteen, October 20

- Comments on midterm
- How to take square root of 17?
- More fun with gyro demos
- More discussion of the gyro-boat
- The physical pendulum (demo and discussion)
- A brief discussion of exciting themes related to oscillations (with demos): damping; forced oscillations; resonance; coupled oscillators; eigenmodes
- Rotations do not commute! A demonstration with a book
- The Berry's phase – an example with a thumb
- Feynman's demo with a coffee cup that shows that sometimes you need 2x2π rotations to bring a system to its original state
- Nutation: derivation of the nutation equations

Lecture sixteen, October 22

- More on Nutation: derivation of the nutation equations, solution, limiting cases
- <u>Fluids</u>: liquids and gasses
- The concept of pressure
- Atmospheric pressure: demo with pumped-out sphere
- Connected vessels (demo)
- More on independence of pressure on the shape of the container: demo with beaker inside a beaker
- The Archimedes' principle: derivation and demo which allowed us to measure the density of copper and aluminum
- Hydrostatics vs. hydrodynamics: demos with pressure gradient in a pipe with uniform cross-section and a pipe with variable cross-section. This is a lead-in to our forthcoming discussions of viscosity and the Bernoulli's principle

Lecture seventeen, October 27

- An easy way to find the center of mass of objects (sliding fingers)
- Demo of simple motion of the center of mass of a dumbbell
- Tides: a qualitative discussion and derivation. Comparison of the effects of the Sun and the Moon. Debunking theories of the Great Flood
- Why many books have a wrong picture of tides: driven oscillations with a frequency above the natural frequency of an oscillator. Demo
- Driven coupled oscillators; normal modes (demo)
- Weight of air: (failed) demo and calculation
- Air pressure: collapsing-can demo

Lecture eighteen, October 29

- How to figure out if your tower will fall? (Demo and discussion: need to drop a perpendicular from the c.m. and see if it falls within the area of the base)
- Transformation of frames: inertial and non-inertial; Einstein's Equivalence Principle
- Energy and momentum for a ball bouncing off a wall as seen indifferent frames. Energy is not invariant with respect to Galilean transformation, neither is whether there is energy exchange between the ball and the wall or not. Total energy is, however, conserved in either frame
- Fictitious forces in non-inertial frames: a cylinder on accelerating table
- Fictitious forces in uniformly rotating frames
- Successful (!) weight of air demo
- Various demos of the Bernoulli's principle as applied to air flow: wind tunnel, Pitot tube, a ball trapped in air flow; roofs and hurricanes

Lecture nineteen, October 30

- **Fluids**. Note: there is very little on fluids in the K&K book. We will start with the simplest case of incompressible and inviscid fluid
- Pressure of liquid at a given depth. Hydraulic lift
- Archimedes' Law; buoyancy

Lecture twenty, November 3

- Potential energy of fluid under pressure – the entire static liquid is equipotential
- Liquid flow in pipes. The continuity equation. Energy conservation → the Bernoulli equation
- The speed with which water flows from a bucket if a whole is punched in the side
- Demos on pressure: Cartesian divers, vacuum gun, model of a rotary-vane pump, hot-air balloon; estimate of the lifting force of the balloon
- Viscosity (for water at room temperature, $\eta \approx 10^{-3}$ in SI units), viscous drags; balls, bubbles, etc.

Lecture twenty one, November 5

- The Stokes formula for a sphere and for a bubble; the concept of attached mass
- The Poiseuille flow
- Viscosity demos; turbulence demos
- Turbulence; the Reynolds number
- Demos that failed last time: isotropy of pressure; model of lungs

Lecture twenty two, November 10

- Frequency of oscillations of water in a cup
- Deep-water gravity waves
- General properties of waves: λ, ω, k; dispersion relation $\omega(k)$; Phase velocity $v=\omega/k$
- String theory; speed of wave on a string; wave equation
- Stroboscope and waves on a rope demo

Lecture twenty three, November 12

- More on string oscillations; standing waves; harmonics
- Wave packets and group velocity $v_g=d\omega/dk$
- Kelvin wake
- Capillary forces, surface tension; pressure under curved surface ($\Delta p=2\sigma/R$). For water at room temperature, $\sigma\approx 73$ mN/m
- Capillary/wetting demos

Lecture twenty four, November 17

- Capillary pressure in the case of unequal curvatures
- Critical size: when are capillary forces important?
- Wetting; contact angle
- Calculation of the height to which fluid rises in a capillary
- Can transport of water to the top of tall trees be explained by capillary forces?
- Capillary waves; dispersion relation, group and phase velocities
- Transverse and longitudinal waves (with demos); how do we know the Earth has a molten core?
- Reflection of waves from fixed and free boundaries (with demos)

Lecture twenty five, November 19

- Estimate of the cut-off wavelength below which capillary effects are more important for waves than gravity, and above which, the opposite is true
- Generalized dispersion relation for capillary-gravity waves
- Wave-particle duality and the quantum-mechanics connection
- A bit about sound waves
- Speed of sound (demo)
- Interference of sound waves (demo)

Lecture twenty six, November 24

- Physics in the bathtub (demo)
- Damped oscillator: physical meaning and general solution
- Theory of forced driven harmonic oscillations with damping
- Another resonance demo
- The Doppler effect: theory + demo

Lecture twenty seven, December 1

- Guest lecture: Prof. Holger Müller, The Physics of Flight

Lecture twenty eight, December 3

- The physics of music (with fire and drumming)

Lecture that was not meant to be this semester

- Oscillations: More on damped oscillator. The Q factor
- Fluids: Rotational flow and vortex motion
- A discussions of the seasons (with a demo); inclination of the Earth rotation axis with respect to ecliptic
- Precession of the Equinoxes
- Derivation (using the equipotential method) and demonstration of the parabolic surface shape of water in a rotating bucket
- **Central-force motion**: general properties
- Centrifugal barrier, effective potential
- General equations of motion; trajectory
- Planetary motion; Physlet computer simulations
- More on planetary motion. **End of central force motion**