Homework # 1; due Thursday, Sept. 3

1. The latitude of San Francisco is approximately 38º North, and that of Moscow is approximately 55º North. If you are on a non-stop flight between these cities, at some point you, most likely, will be flying above the polar circle (66º North), and sometimes nearly over the North Pole. Why is such a route chosen by the airlines?

2. Destroyer ship "Aurora" is heading South-East at a constant speed of 30 knots. At noon, passenger yacht "Pegasus" is found 200 nautical miles directly south of "Aurora" heading North-East at 20 knots.

    a) Assuming both ships maintain their heading and speed, determine the time at which the separation between them is minimal.
    b) What is this minimal separation?
    c) Discuss whether the approximation that the Earth is (locally) flat is adequate for this problem.

    Hints: There is a clever trick (that was known to Galileo) that allows solving this problem in a straightforward way. You might have to look up definitions of a nautical mile and a knot.

3. Short exercises from K&K: 1.1, 1.2, 1.4, 1.5, 1.7

4. A research airplane, which is a rented commercial Boeing-737, takes off from San Francisco and flies due West, landing only for brief refueling/maintenance, eventually completing two full circles along the Earth's parallel. Sketch the local time on the ground vs. the time in San Francisco. State what assumptions you are making, for example, what do you take for the airplane's speed and why.

Homework # 2; due Thursday, Sept. 10

1. Watch the NASA "educational" video found at: http://space.about.com/library/weekly/blvidgalileopisa.htm describing Galileo's famous leaning tower of Pisa experiment. How many errors can you find in the narration? List the errors and suggest corrections.

2. The most accurate atomic clocks today (for example, the one at the National Institute of Standards and Technology at Boulder, CO) incorporate what's called an atomic fountain, a device in which atoms that have been laser-cooled to near-zero temperatures are tossed vertically in a vacuum chamber. They proceed to fly up, but eventually turn around and fall due to gravity. To make the clock as accurate as possible, it is desirable to maximize the time the atoms spend in their ballistic journey (I will be happy to explain why this is, but this is a bit outside of H7A).

    a. Approximately how long does it take the atoms to return to the bottom of the apparatus, if the height of the vacuum chamber is 3 m?
    b. The NIST clock uses cesium (Cs) atoms with atomic weight of 133. What would be the return time for rubidium (Rb) atoms with atomic weight 87?
    c. Why do people talk about setting up atomic fountain clocks in space?

3. K&K problems: 1.11, 1.13, 1.14, 1.16, 1.18 (note: since acceleration is a vector quantity, so is jerk. Find the vector expression for jerk.), 1.21
4. Consider a point on the surface of a rigid wheel of radius R rolling on the ground with speed v (without sliding). Find analytical expressions for $x(t)$, $y(t)$, $y(x)$, $v_x(t)$, and $v_x(t)$ for this point and sketch the plots of these quantities. Note that the $y(x)$ plot represents the trajectory of the point. For the plots representing time dependences, it is convenient to place them one above the other, so they all have a common time axis. This helps visualize what's going on at any given time. Note: if you prefer to express the trajectory as $x(y)$ instead of $y(x)$, this is OK!
5. Please feel free to give the Instructor and the GSI feedback on homework. Is there too much or too little of it? Are the problems too hard or too easy?

Homework # 3; due Thursday, Sept. 17

1. The radius of Mars is about two times smaller than that of the Earth. Using this fact, what can you say about its mass of the red planet and the acceleration due to gravity at the planet's surface (compared to these quantities for the Earth)? What additional assumptions have you used for these estimates?
2. K&K problems: 2.4, 2.8, 2.10, 2.17, 2.18, 2.22, 2.26, 2.27
3. Optional: try the challenge problems given in class.
    a. What is the optimal shape of a blob of play-dough of a fixed total mass and density to maximize gravitational pull at a point?
    b. Cowboy with a lasso on a conical ice mountain: what is the largest angle of the cone for the poor guy to survive?

Homework # 4; due Thursday, Sept. 24

1. A fisherman is sitting at the stern of a boat, and both the boat and the man are at rest. The fishermen than moves towards the bow, and eventually, the boat and the man are at rest again. Find the displacement of the boat assuming that the motion of the boat in the water is turbulent, i.e., it is characterized by a friction force that is $\propto -v^2$, where v is the velocity of the boat with respect to water. You may assume some simple model for the process if necessary (for example, the man jumps, and after a bit of a ballistic flight, lands on the bow).
2. K&K problems: 3.2, 3.4, 3.7, 3.9, 3.12, 3.14, 3.15, 3.20 (optional, extra credit)

Homework # 5; due Thursday, Oct. 1

1. (Extra credit) It is known that the Coriolis forces are important in atmospheric phenomena such as typhoons, affecting the direction of rotation in the atmospheric vortices depending on the hemisphere and the direction of the vortices' motion. It has been also suggested that Coriolis forces may be affecting the direction of rotation in the familiar bathtub vortices appearing when you pull the bathtub plug. In this problem, you are asked to estimate the order of magnitude of the Coriolis forces in a bathtub, and argue whether or not they are indeed important.
    a) Consider an element of water in the bathtub. Estimate its velocity. Is it the same for all elements of water? Which elements are important in this problem?
    b) Using the angular velocity of the Earth's rotation and the results of part a), estimate the magnitude of the Coriolis acceleration.
    c) In order to decide whether the Coriolis forces are important or not, we need to compare them to some other forces acting on water elements. Let us consider just one of such forces (feel free to suggest others!): the force due to imperfect shape of the bathtub. Indeed, if we imagine water going down a curled hose, then, certainly, water will be rotating in the direction given by the shape of the curl. It is hard to imagine that a bathtub would be manufactured without defects at a level of at least 1 mm/per 1 m of the tub length. Estimate the asymmetric accelerations due to these imperfections. How do they compare to the Coriolis acceleration?

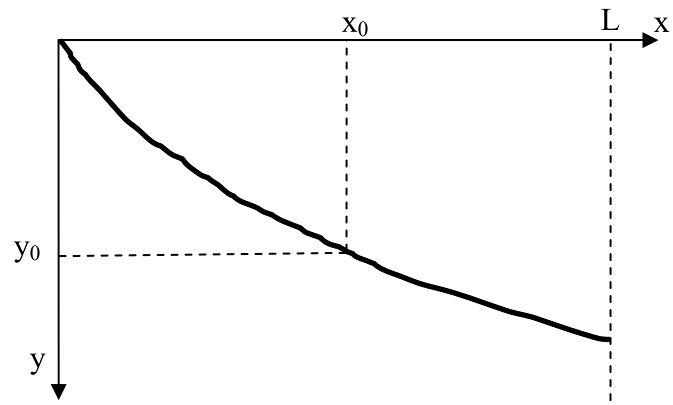
d) Regardless of the Coriolis forces, can you explain why the bathtub vortices are formed?

2. (Extra credit) A Foucault pendulum is used to demonstrate the fact that the Earth rotates around its axis: the pendulum maintains the plane in which it oscillates, so the oscillation plane is rotating with respect to the room where it is set up (as the room is affixed to the Earth). There is a nice Foucault pendulum that can be seen at the California Academy of Sciences: http://www.calacademy.org/products/pendulum.html. Determine the period of the apparent rotation of the oscillation plane for this particular pendulum.

3. (Extra credit) In the fourth problem of HW-2, you have derived equations describing the trajectory of a point on the surface of a rolling wheel, the cycloid. Let us parameterize a portion of the cycloid in the following way:

$$x(\varphi) = \frac{L}{\pi}(\varphi - \sin\varphi)$$
$$y(\varphi) = \frac{L}{\pi}(1 - \cos\varphi)$$

, where we have placed the beginning of the curve in the origin, φ runs from 0 to π, and $x(\varphi)$=L. Now let us suppose that the curve is actually made out of wire, which is positioned as shown in the Earth gravitational field (the y axis is pointing down), and that a bead placed on the wire can slide without friction. Suppose we release the bead with zero initial velocity at a point that is given by $\varphi=\varphi_0$ (with coordinates x_0,y_0).



- Determine the time it will take the bead to reach the end of the wire ($\varphi=\pi$).
- Examine the dependence of the time on φ_0. If you have solved the problem correctly, you have discovered the special property of the cycloid called tautochronism. Explain what's going on in physical terms.

Hint: you may (or may not) find the following integral useful:

$$\int_{\varphi_0}^{\pi}\left(\frac{1-\cos\varphi}{\cos\varphi_0 - \cos\varphi}\right)^{1/2} d\varphi = \pi .$$

Homework # 6; due Thursday, Oct. 15

1. (Extra credit) Devise and describe a microscopic model for solid friction that will reproduce the following features that are observed (to a certain degree) in the experiments:
   a. Maximum static friction force is proportional to the normal force

   b. Maximum static friction is independent of area
   c. Dynamic friction is less than static friction

   You may find it useful to invoke the Hooke's law. The solution of this problem should take the form of a brief (no more than one page) essay. Use equations and drawings as needed. High quality of scientific writing is important, including clarity, grammar, spelling, etc.
2. K&K problems: 4.3, 4.6, 4.11, 4.12, 4.13, 4.21, 4.25, 4.30

Homework # 7; due Thursday, Oct. 22

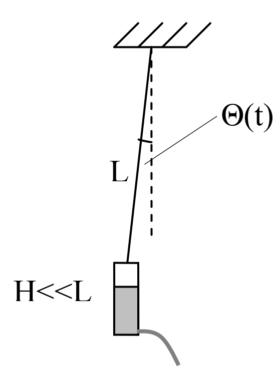
1. K&K problems: 4.5, 4.7, 4.14 Calculate the numerical value of ω (in rad/s) for M=1 kG, a=5 cm, 4.29, 4.23
2. (Extra credit; we will not dwell on this Ch. 5 material that much, although it is very useful) 5.1, 5.3, 5.7
3. K&K problems: 6.2, 6.13

Homework # 8; due Thursday, Nov. 5

1. K&K problems: 7.1, 7.3, 7.4, 7.5, 7.6 (do not forget to consider limiting cases), 7.9 (solve this problem and debunk a common misconception!)
2. Imagine that you are asked to give a guest lecture for the Physics H7a class on the subject of torque-free precession. Research (using books, internet, personal connections…) and briefly describe three impressive examples to illustrate what this is.
3. A whimsical piece of furniture has the base shaped like a star as shown. Formulate the condition in terms of the location of the center of mass of the object that the piece would not topple over. A sketch would be helpful.

4.
   a. Derive the expression for the acceleration in the laboratory frame of a small object located at a position $\boldsymbol{R}_r$ and moving with a (not necessarily constant) velocity $\boldsymbol{v}_r$, both with respect to a platform rotating with angular velocity $\boldsymbol{\omega}$.
   b. If we want to work in the non-inertial rotating frame, we need to assume that there are fictitious forces acting on the body. What are these forces? Discuss their magnitude and the direction based on your derivation of the acceleration.
   c. Please revisit your solution to the midterm problem regarding the displacement of a marble dropped from the Empire State Building in light of your findings.

Homework # 9; due Thursday, Nov. 12



1. Calculate the Archimedes' force due to air acting on your body.
2. A cylindrical cup of diameter R and height H is filled with water to the top. The cup is attached to a long string of length L as shown. A hole of diameter δ is punctured near the bottom of the cup, and as the water starts to escape the cup at t=0, the system is released in its equilibrium position (so the cup does not swing

around as a pendulum). Determine the angle that the string makes with the vertical Θ(t). Neglect viscosity. Assume Θ(t)<<1.

3. A cylindrical barrel of diameter D=1 m and height h=1 m is filled with water to the top. The water is at room temperature of 20°C. Near the bottom of the barrel, there is a tap that connects to a hose with inner diameter d=1 cm. The hose has length L=10 m and is lying flat on the ground.
    a. Assuming that the speed with which water escapes the barrel once the tap is opened is determined by the viscous forces, what is the height of the water in the barrel as a function of time. Assume that the tap is opened at t=0.
    b. How long does it take to empty the barrel?
    c. Same as a) and b), except for the case of no hose attached to the tap.
    d. In parts a) and b) (long hose), viscosity was important, while in part c) (no hose or a very short one) it was not. This means that there is some "cut-off" length of the hose (of the given diameter), L^*, which marks the onset of the regime where the dynamics is dominated by viscosity. Estimate L^*.
    e. Which of the above results would change (and how) if water was not at room temperature, but just above the melting point of 0°C.
4. Extra credit: A brick falls flat onto a tennis ball resting on the ground and bounces back to the height of h = 1 m. What height will the ball bounce to? Make reasonable assumptions, for example, neglect the size of the ball compared to the bounce height.

Homework # 10; due Thursday, Nov. 19

1. A bubble of air which is 1 mm in diameter is released without initial velocity in the volume of glycerol at room temperature. Describe the motion of the bubble as a function of time assuming that its diameter remains unchanged. Note that friction force acting on the bubble is given by

$$\vec{F}_{fr} = -4\pi\eta R\vec{v},$$

where η is the viscosity of the liquid, R is the radius of the bubble, and $\vec{v}$ is its velocity. Note that the numerical coefficient in this formula is different from that for a solid sphere moving in the liquid (which is 6π). Explain qualitatively why the numerical coefficients in the viscous drag (friction) formulae for a solid sphere and a bubble are different.

2. A thin metal sheet of square shape with dimensions of 1 cm by 1 cm is immersed in deep water (at room temperature) far away from the surface, the bottom, and the sides of the container and is towed parallel to its plane with constant speed of 1 cm/s.
    a. Can the water flow past the sheet be considered laminar? (In order to answer this question, evaluate the Reynolds number.)
    b. Write the analytical expression for the friction force that is accurate up to a numerical factor of order unity.
    c. Plug in numbers into the expression obtained in (b).
3. Extra credit: Have you wondered how can it happen that the friction force in a fluid is proportional to the first power of velocity of the body with respect to the fluid at low velocities, and to the square of the velocity at high relative velocities? Here is a model that *might* illuminate this. Consider a thin flat plate of area *A* moving in a gas with velocity *v* perpendicular to the plane of the plate. Assume that the gas consists of

molecules of mass m with number density n (measured in molecules per cubic meter) that do not interact with each other, and are all moving with thermal speed v_T , which, for simplicity, we assume the same for all molecules; however randomly directed. We will also assume that when a molecule hits the plate, it undergoes elastic scattering.

a. First assume that the plate is stationary, v=0. How many molecules hit the plate from each side per unit time?
b. What is the gas-pressure force pushing on the plate from either side?
c. Now suppose the plate is moving with $v<<v_T$. Calculate the friction force, which is the difference in the pressure forces from the two sides.
d. Repeat this for $v>>v_T$. This is a simpler problem as we can assume the molecules of the gas stationary before the plate hits them.
e. Connect the results of you calculations to the v vs. v^2 question.

Homework # 11; due Thursday, Dec. 3

1. In class, we have discussed that in the modern-day view of Nature, particles (for example, electrons), under certain conditions, display properties normally associated with waves, while waves (for example, light) may display properties normally associated with particles. Since the notions used to describe waves, such as frequency, wave vector, dispersion relations, etc. are quite universal, by studying waves on the surface of water; we have actually learned something about other types of waves as well. In this problem, we will illustrate this by briefly discussing probability-amplitude waves corresponding to electrons that are normally studied in quantum mechanics courses.
   a. The dispersion relation for a free-electron wave is: $\omega = \frac{\hbar}{2m} k^2$. Here ω and k are the frequency and the wave vector of the wave, respectively; $\hbar$ is a constant (the Planck's constant); and m is the mass of the electron. What are the SI units of $\hbar$?
   b. What are the phase and group velocities of the electron wave? What is the relation between the two velocities? How does this relation compare to such relations for other types of waves that you know (water waves, sound, light, etc.)?
   c. In quantum mechanics it is shown that the energy of the electron is $E=\hbar\omega$, and the magnitude of its momentum is $p=\hbar k$. Rewrite the dispersion relation in terms of energy and momentum. Does it look familiar?
   d. Based on part (c), determine which of the two velocities of the electron wave, phase and group (if any) corresponds to the electron's classical velocity?
2. Explain how water striders (bugs that run on the water surface) are supported by surface tension. Make estimates proving that surface tensions forces are sufficiently strong for this.
3. Why do we use soap when we want to make bubbles?