DRAW THE FREE BODY DIAGRAMS:
\[ T = m a_m \]
\[ M g - T = M a_M \]
\[ \tau = TR = I \alpha \]
where \( I = \frac{1}{2} MR^2 \).

THE CONSTRAINT IS THAT THE LENGTH OF THE STRING IS ONLY INCREASED BY THE "ROLLING" OF THE DISK, WHICH IMPLIES:
\[ a_M = a_m = \alpha R \]

EQUATING EXPRESSIONS FOR THE TENSION \( T \):
\[ M a_m = M g - M a_M \quad \text{AND} \quad M a_m = \frac{I \alpha}{R} \]
\[ M a_m = \frac{1}{2} MR \alpha \]

THE CONSTRAINT EQUATION GIVES US:
\[ m a_m = M g - 2(m + \frac{1}{2} M) a_m \]
\[ 3 \cdot m a_m + M a_m = M g \]

\[ a_m = \left( \frac{M}{M + 3m} \right) g \]
\[ m a_m = \frac{1}{2} MR \alpha \]
\[ \alpha = \frac{2m}{M} \frac{1}{R} a_m \]
\[ a_M = \alpha R + a_m \]
\[ a_M = \left( \frac{M + 2m}{M + 3m} \right) \frac{g}{R} \]

\[ T = m a_m \]
\[ T = \left( \frac{m M}{M + 3m} \right) g \]
TORQUE EQN: \[ \frac{mg L}{2} \sin \theta = \frac{1}{3} mL^2 \dot{\theta} \]

\[ \theta \]

\[ m \] \[ \Rightarrow \] \[ \text{mg} \sin \theta \]

ENERGY CONSERVATION: \[ E = \frac{1}{2} I \dot{\theta}^2 + mg h \]

INITIAL ENERGY \[ I = \frac{1}{3} mL^2 \]

(bar pivoted from end)

\[ E = \frac{mg L}{2} = \frac{1}{2} \frac{1}{3} mL^2 \dot{\theta}^2 + mg \frac{L}{2} \cos \theta \]

\[ \frac{1}{6} mL^2 \dot{\theta}^2 = \frac{mg \frac{L}{2}}{2} (1 - \cos \theta) \]

AT \( \theta = 90^\circ \) THE PIVOT MUST ACT ON THE BAR TO HOLD IT IN A UNIFORM CIRCULAR PATH:

\[ F_x = -\frac{mL}{2} \dot{\theta}^2 \]

\[ \frac{1}{6} mL^2 \dot{\theta}^2 = \frac{mgL}{2} \]

FROM ENERGY CONSERVATION.

\[ F_x = -\frac{3}{2} mg \]

\[ \frac{mL}{2} \dot{\theta}^2 = \frac{3}{2} mg \]

ALSO, THE PIVOT CAN PROVIDE A FORCE IN THE \( y \)-DIRECTION:

\[ F_y - mg = m \ddot{y} \]

\[ m \ddot{y} = m \frac{L \dot{\theta}^2}{2} = \frac{3}{4} mg \]

\[ F_y = \frac{mg}{4} \]
WE CAN USE FICTITIOUS FORCES TO SOLVE THIS PROBLEM.

THE ACCELERATION OF THE BLOCK IS GIVEN BY THE SUM OF THE FORCES Acting ON IT:

\[ M \ddot{a}_M = kx - f \]

\[ a_M = \frac{Kx - f}{M} \]

WE CAN THEN WORK IN THE ACCELERATING REFERENCE FRAME OF THE BLOCK, WHERE THERE IS THE ADDITIONAL FICTITIOUS FORCE \( F_{fic} \) Acting ON THE WHEEL:

\[ F_{fic} = -Ma_M = \frac{M}{M} (f - kx) \]

THE FORCES ACTING ON THE WHEEL ARE:

\[ M \ddot{x} = f - kx + \frac{M}{M} (f - kx) \]

\[ \begin{array}{c}
      \hline
      \text{ASSUME WHEEL IS DISK}
      \hline
    \end{array} \]

\[ f = -\frac{1}{2}m \dot{x} \]

\[ m \ddot{x} = \left(1 + \frac{m}{M}\right)(f - kx) \]

\[ \left(\frac{m}{1 + m/M}\right) \ddot{x} + \frac{1}{2}m \dddot{x} = -kx \]

\[ \omega^2 \dot{x} + \left(\frac{k}{\frac{1}{2}M + \frac{m}{1 + m/M}}\right)x = 0 \]