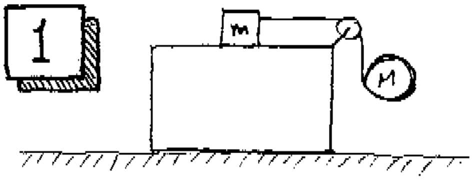
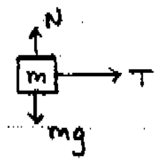
1



DRAW THE FREE BODY DIAGRAMS:



$$T = ma_m$$

$$Mg - T = Ma_M$$

$$\tau = TR = I\alpha$$

where $I = \frac{1}{2}MR^2$.

THE CONSTRAINT IS THAT THE LENGTH OF THE STRING IS ONLY INCREASED BY THE "ROLLING" OF THE DISK, WHICH IMPLIES:

$$a_M - a_m = \alpha R$$

EQUATING EXPRESSIONS FOR THE TENSION T:

$$ma_m = Mg - Ma_M \qquad \text{-AND-} \qquad ma_m = \frac{I\alpha}{R}$$

$$ma_m = \frac{1}{2}MR\alpha$$

THE CONSTRAINT EQUATION GIVES US $\Rightarrow$

$$ma_m = \frac{1}{2}M(a_M - a_m)$$

$$\left(m + \frac{1}{2}M\right)a_m = \frac{1}{2}Ma_M$$

$$ma_m = Mg - 2\left(m + \frac{1}{2}M\right)a_m$$

$$3ma_m + Ma_m = Mg$$

(A) $$\boxed{a_m = \left(\frac{M}{M+3m}\right)g}$$

$$ma_m = \frac{1}{2}MR\alpha$$

$$\alpha = \frac{2m}{M}\frac{1}{R}a_m$$

$$a_M = \alpha R + a_m$$

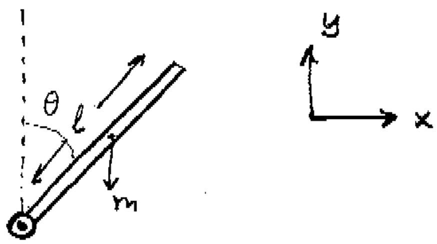
(A) $$\boxed{a_M = \left(\frac{M+2m}{M+3m}\right)g}$$

$$\boxed{\alpha = \left(\frac{2m}{M+3m}\right)\frac{g}{R}}$$ (B)

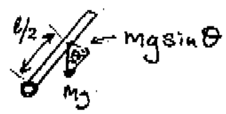
$$T = ma_m$$

$$\boxed{T = \left(\frac{mM}{M+3m}\right)g}$$ (C)

2



TORQUE EQN: $\frac{mg\ell}{2}\sin\theta = \frac{1}{3}m\ell^2\ddot{\theta}$



ENERGY CONSERVATION: $E = \frac{1}{2}\cdot I\dot{\theta}^2 + mgh$

$I = \frac{1}{3}m\ell^2$ (bar pivoted from end)

height of CM $= \frac{\ell}{2}\cos\theta$

INITIAL ENERGY

$$E = \frac{mg\ell}{2} = \frac{1}{2}\,\frac{1}{3}m\ell^2\dot{\theta}^2 + mg\frac{\ell}{2}\cos\theta$$

$$\frac{1}{6}m\ell^2\dot{\theta}^2 = \frac{mg\ell}{2}(1-\cos\theta)$$

AT $\theta = 90°$ THE PIVOT MUST ACT ON THE BAR TO HOLD IT IN A UNIFORM CIRCULAR PATH:

$$F_x = -m\frac{\ell}{2}\dot{\theta}^2$$

$$\boxed{F_x = -\frac{3}{2}mg}$$

$$\frac{1}{6}m\ell^2\dot{\theta}^2 = \frac{mg\ell}{2}$$

FROM ENERGY CONSERVATION.

$$\frac{m\ell}{2}\dot{\theta}^2 = \frac{3}{2}mg$$

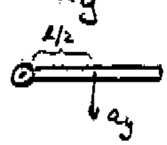
ALSO, THE PIVOT CAN PROVIDE A FORCE IN THE y-DIRECTION:

$$F_y - mg = ma_y$$

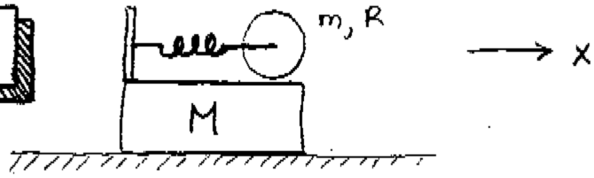
$$\boxed{F_y = \frac{mg}{4}}$$

$$ma_y = m\frac{\ell\ddot{\theta}}{2} = -\frac{3}{4}mg$$

FROM TORQUE EQN. w/ $\theta = 90°$



3

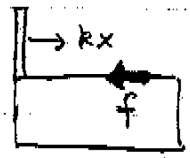


WE CAN USE FICTIONAL FORCES TO SOLVE THIS PROBLEM.

THE ACCELERATION OF THE BLOCK IS GIVEN BY THE SUM OF THE FORCES ACTING ON IT:

$$M a_M = kx - f$$

$$a_M = \frac{kx - f}{M}$$

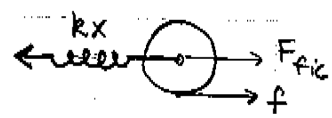


WE CAN THEN WORK IN THE ACCELERATING REFERENCE FRAME OF THE BLOCK, WHERE THERE IS THE ADDITIONAL FICTIONAL FORCE F_{fic} ACTING ON THE WHEEL:

$$F_{fic} = -m a_M = \frac{m}{M}(f - kx)$$

THE FORCES ACTING ON THE WHEEL ARE:

$$m\ddot{x} = f - kx + \underbrace{\frac{m}{M}(f - kx)}_{F_{fic}}$$



ALSO WE HAVE TORQUE ON THE WHEEL FROM FRICTION f, (FICTITIOUS FORCE APPLIES NO TORQUE, ACTS ON CM).

$$fR = I\ddot{\theta}$$

$$= \frac{1}{2} mR^2 \ddot{\theta}$$ ASSUME WHEEL IS DISK

$$= -\frac{1}{2} mR\ddot{x}$$

FOR ROLLING w/o SLIPPING

$$\ddot{x} = -R\ddot{\theta}$$

$$f = -\frac{1}{2} m\ddot{x}$$

$$m\ddot{x} = \left(1 + \frac{m}{M}\right)(f - kx)$$

$$m\ddot{x} = \left(1 + \frac{m}{M}\right)\left(-\frac{1}{2}m\ddot{x} - kx\right)$$

$$\left(\frac{m}{1 + m/M}\right)\ddot{x} + \frac{1}{2}m\ddot{x} = -kx$$

SHO EQN.

$$\ddot{x} + \underbrace{\left(\frac{k}{\frac{1}{2}m + \frac{m}{1 + m/M}}\right)}_{\omega^2} x = 0$$