Cavity quantum electrodynamics with solid state spin ensembles and superconducting resonators

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Physics H190, Spring 2011
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Cavity QED

-atom in a cavity
Cavity QED

- atom in a cavity

- ensemble of spins coupled to a superconducting resonator
Motivation

- Collective coupling → strong coupling
- Long coherence times
- Swap quantum state between a superconducting qubit and the spin ensemble → quantum memory
- Access new regimes of cavity QED?
Outline

- Cavity QED

- Superconducting resonators

- Coupling NV centers to a resonator

- Outlook
Cavity QED

CHAPTER 2. CAVITY QUANTUM ELECTRODYNAMICS

Figure 2.1: A two level atom interacts with the field inside of a high finesse cavity. The atom coherently interacts with the cavity at a rate, \( g \). Also represented are decoherence processes that allow the photon to decay at rate \( \kappa \), the atom to decay into modes not trapped by the cavity, and the rate at which the atom leaves the cavity, \( 1/T \). To reach the strong coupling limit the interaction strength must larger than the rates of decoherence \( g > \kappa, \gamma, 1/T \) transit.

Leakage or absorption by the cavity results in a photon decay rate, \( \kappa \), sometimes expressed in terms of the quality factor \( Q = \omega/\kappa \). Often, and particularly in circuit QED, \( \kappa \) is set by the desired transparency of the mirrors to allow some light to be transmitted to a detector as a means of probing the dynamics of the system. In the absence of other decay mechanisms, the radiative decay of the atom can be completely described in terms of the coherent interaction with the cavity and decay of cavity photons (which are measured). This very well modeled (and often slow) decay makes cavity QED a good test-bed for studying quantum measurement and open systems [Mabuchi2002].

In practice, the atom may also decay at rate, \( \gamma \), into non-radiative channel so radiation modes not captured by the cavity. Finally in atomic implementations, the atoms have a finite lifetime or transit time \( T \) transit, before exiting the cavity. The competition between coherent and incoherent processes is most evident when the atom is resonant with the cavity, and the two systems can freely exchange energy. In the absence of decay, an excitation placed in the system will coherently oscillate between an atomic excitation and a photon in the cavity. This is often called a vacuum Rabi oscillation because it can be interpreted as vacuum fluctuations which stimulate photon emission and absorption by the cavity and atom from the cavity.

When many oscillations can be completed before the atom decays or the photon is lost, the system reaches the strong coupling limit of cavity QED (\( g > \gamma, \kappa, 1/T \) transit).
Cavity QED

Jaynes-Cummings Hamiltonian

\[ H_{JC} = \hbar \omega_r (a^\dagger a + 1/2) + \hbar \frac{\omega_a}{2} \sigma_z + \hbar g (a^\dagger \sigma^- + a \sigma^+) \]
Here and discussed in more depth in Section 3.3.

Describes a dipole interaction to be henceforth referred to as a spin, qubit, atom, or simply two-level system.

This coherent behavior of this coupled system is described by the Jaynes-Cummings Hamiltonian, where the first term represents the energy of the electromagnetic field and the second term represents the atom as a spin-1/2 with transition energy, \( \hbar \omega \).

In practice, the atom may also decay at rate, \( \gamma \), and the rate at which the atom leaves the cavity, \( 1/T_{\text{transit}} \).

This Hamiltonian, circuits, and techniques are general and could be applied to any physical system, including superconducting circuits. While specific to cavity QED, the techniques are general and could be applied to any physical system.

The Jaynes-Cummings Hamiltonian is valid in all experiments performed using the Jaynes-Cummings approximation, \( \kappa > \gamma > \omega_c > \sqrt{\gamma \kappa} \).

When many oscillations can be completed before the atom decays, \( T_\text{transit} \) becomes high, and certain approximations are often made.

In the absence of other decay mechanisms, the radiative decay of cavity photons (which are measured) is a good test-bed for studying quantum measurement in open systems.

Cavity QED also considers phenomena that obscure the evolution of the system. In the absence of other decay mechanisms, the radiative decay of cavity photons (which are measured) is a good test-bed for studying quantum measurement in open systems.

Jaynes-Cummings Hamiltonian

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Cavity
**Jaynes-Cummings Hamiltonian**

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Cavity \hspace{20mm} Atom
Jaynes-Cummings Hamiltonian

\[ H_{JC} = \hbar \omega_r (a^\dagger a + 1/2) + \hbar \frac{\omega_a}{2} \sigma_z + \hbar g (a^\dagger \sigma^- + a \sigma^+) \]

Cavity QED
Dispersive limit: $\Delta \equiv \omega_a - \omega_r \gg g$

$$H \approx \hbar \left( \omega_r + \frac{g^2}{\Delta} \sigma_z \right) (a^\dagger a + 1/2) + \hbar \omega_a \sigma_z / 2$$
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$|0\rangle \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$\omega_r (|0\rangle) + 2g^2 / \Delta = \omega_r (|1\rangle)$$

$|1\rangle \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix}$
Superconducting Microwave Resonators

Transmission line resonator

\[ \omega_r (|0\rangle + 2g^2/\Delta) = \omega_r (|1\rangle) \]
Collective coupling to spins
Collective coupling to spins

- Magnetic fields in the resonator interact with the spins
- Store a single resonator photon in a collection of spins
- Coupling strength scales as $\sqrt{N}$

Strong Coupling

\[ g \gg \kappa, \gamma \]
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\[ \kappa = \frac{2\pi f}{Q} \approx 1 \text{ Mhz, for } f = 2.7 \text{ GHz and } Q \approx 10,000 \]
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single spin: \( g \approx 10 \text{ Hz} \)
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\[ 10^{12} \text{ spins: } \sqrt{N} \times g \approx 10 \text{ MHz} \]
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How do we ‘see’ the spins?

- Tune the transition frequency across the resonator frequency
- Look for an avoided crossing
How do we ‘see’ the spins?

- Tune the transition frequency across the resonator frequency

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![Diagram showing no coupling between spins and resonator as a function of magnetic field and frequency.](image-url)
How do we ‘see’ the spins?

- Tune the transition frequency across the resonator frequency

- Look for an avoided crossing
How will the spins tune with $\mathbf{B}$?

How will the spins tune with $B$?

How will the spins tune with $B$?

Some results!

<table>
<thead>
<tr>
<th>Frequency (GHz)</th>
<th>B (mT)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.68</td>
<td>-10</td>
</tr>
<tr>
<td>2.69</td>
<td>-9</td>
</tr>
<tr>
<td>2.70</td>
<td>-8</td>
</tr>
<tr>
<td>2.71</td>
<td>-7</td>
</tr>
<tr>
<td>2.72</td>
<td>-6</td>
</tr>
<tr>
<td>2.73</td>
<td>-5</td>
</tr>
</tbody>
</table>

\( \varphi = 45^\circ \)

\[ |S_{21}| \text{ (dB)} \]
Some results!

- [Image -129x135 to 869x604]

- 

\begin{align*}
\text{Frequency (GHz)} &: 2.68, 2.69, 2.7, 2.71, 2.72, 2.73 \\
\text{B (mT)} &: 0, 5, 10
\end{align*}

- [Image -129x135 to 869x604]
Some results!

**blue:** phi = 45 deg, B = 8.2 mT

\[
\frac{g_I}{2\pi} = 18.5 \text{ MHz}
\]

**red:** phi = 0 deg, B = 14.3 mT

\[
\frac{g}{2\pi} = 26.3 \text{ MHz} = \sqrt{2} \times \frac{g_I}{2\pi}
\]
Experimental Setup
Experimental Setup
Experimental Setup
Experimental Setup
Experimental Setup
What else can you do?

- Use dispersive shift in cavity resonance frequency to characterize spin relaxation time

- Swap experiment: quantum memory

- Other solid state spin systems:
  - Bismuth doped Si
  - Ruby
  - N C$_{60}$
Thanks!