Problem 1

$$m = 939.57\frac{\text{MeV}}{c^2} = \frac{939.57\ \text{MeV}}{\left(299792458\frac{\text{m}}{\text{s}}\right)^2} \times \frac{10^6\ \text{eV}}{\text{MeV}} \times \frac{1.6022 \times 10^{-19}\ \text{J}}{\text{eV}} \times \frac{10^3\ \text{g}}{\text{kg}} = 1.675 \times 10^{-24}\ \text{g}$$

$$E = mc^2 = 939.57\frac{\text{MeV}}{c^2} \times c^2 = 9.3957 \times 10^8\ \text{eV}$$

For a free particle, $E = \frac{\hbar^2 k^2}{2m}$. Set $m = \frac{E}{c^2}$ and solve for k:

$$k = \frac{\sqrt{2mE}}{\hbar} = \frac{\sqrt{2}E}{\hbar c} = \frac{\sqrt{2} \cdot 939.57\ \text{MeV}}{197\ \text{MeV fm}} \times \frac{10^{13}\ \text{fm}}{\text{cm}} = 6.78 \times 10^{13}\ \text{cm}^{-1}$$

$$\omega = \frac{E}{\hbar} = \frac{939.57\ \text{MeV}}{1.055 \times 10^{-24}\ \frac{\text{m}^2\text{kg}}{\text{s}}} \times \frac{10^6\ \text{eV}}{\text{MeV}} \times \frac{1.6022 \times 10^{-19}\ \text{J}}{\text{eV}} = 1.426 \times 10^{24}\ \text{s}^{-1}$$

$$\nu = \frac{\omega}{2\pi} = 2.272 \times 10^{23}\ \text{Hz}$$

$$T = \frac{E}{k_b} = \frac{939.57\ \text{MeV}}{8.6173 \times 10^{-5}\ \text{eV K}^{-1}} \times \frac{10^6\ \text{eV}}{\text{MeV}} = 1.090 \times 10^{13}\ \text{K}$$

$$\lambda = \frac{2\pi}{k} = 9.32 \times 10^{-14}\ \text{cm}$$

Problem 2

From the Journal of Research of of the National Institute of Standards and Technology, Volume 110, Number 3, May to June 2005, there was an article entitled "Search for a Neutron Electric Dipole Moment" by R. Golub and P. R. Huffman. Their research concluded the neutron has an electric dipole moment of no greater than $3 \times 10^{-29}\ e\ \mathrm{cm}$ (although the actual value is unknown). However, the magnetic dipole moment is known to be $-1.91\frac{e\hbar}{2m_p c}$. Therefore:

$$\left|\frac{\text{Electric Dipole Moment}}{\text{Magnetic Dipole Moment}}\right| \leq \frac{3 \times 10^{-29}\ e\ \mathrm{cm}}{1.91\frac{e\hbar}{2m_p c}}$$
$$= \frac{2(3 \times 10^{-29}\ \mathrm{cm})\left(938.28\frac{\mathrm{MeV}}{c^2}\right)c}{1.91(6.582 \times 10^{-16}\ \mathrm{eV\ s})}$$
$$= \frac{2(3 \times 10^{-29})(938.28)}{1.91(6.582 \times 10^{-16})(29979245800)}$$
$$= 1.5 \times 10^{-21}$$

So the ratio of the neutron's electric and magnetic dipole moments is no bigger than 1.5×10^{-21}.

Prob 3

Orders of magnitude for Neutron flux

Chadwick experiment:

There are approximately 30 neutrons emitted from a berylium source per One million alpha particles which corresponds to a multiplier of $3 \cdot 10^{-5}$

Chadwick (1932) explains the that the polonium source is a 1cm^3 diameter disk with a deposit of polonium. The density of polonium is $9 \frac{\text{g}}{\text{cm}^3}$. Assuming negligible thickness ($\sim$ 0.01mm) there would be approximately 0.01g of polonium per cm^2, or about 0.04g of polonium on the silver disk. The radioactivity of polonium is $6.21 \cdot 10^{11} \frac{\text{disintegrations}}{\text{g s}}$ which corresponds to approximately $1 \cdot 10^{10} \frac{\text{disintegrations}}{\text{s}}$. The result is $(3 \cdot 10^{-5}) \cdot (2.5 \cdot 10^{10}) \approx 7.5 \cdot 10^{5} \frac{\text{neutrons}}{\text{s}}$

SNS experiment:

There are approximately 30 neutrons emitted from a lead particle per spallation

The stats of the SNS setup are 1GeV protons in a 2MW beam, which is pulsed in a width of 1us at 60hz. With 2MW average beam power this corresponds $1 \cdot 10^{25} \frac{\text{eV}}{\text{s}} = 1 \cdot 10^{16} \frac{\text{protons}}{\text{s}} \approx 3 \cdot 10^{17} \frac{\text{neutrons}}{\text{s}}$.

Comparison:

By comparing $7.5 \cdot 10^{5} \frac{\text{neutrons}}{\text{s}}$ to $3 \cdot 10^{17} \frac{\text{neutrons}}{\text{s}}$ conclude that the SNS source has around 11-12 orders of magnitude greater flux than the Chadwick source, assuming the targets (and beams) are of similiar size.

This number agrees with the number given in the wikipedia article on neutron sources which cites $3 \cdot 10^{5} \frac{\text{neutrons}}{\text{s}}$ for a simple source and $1 \cdot 10^{17} \frac{\text{neutrons}}{\text{s}}$ for a high efficiency spallation source like the SNS. Which is a similiar 12 magnitude conclusion that we came to.

PROBLEM 6

The neutron reproduction factor k tells us the average number of neutrons from each fission reaction that causes a subsequent fission. With this definition in mind, if we initiate our fission chain with a single neutron and if n generations have passed, then $\sum_{i=0}^{n} k^i$ atoms have undergone fission. Note that since this is a geometric sum, $\sum_{i=0}^{n} k^i = \frac{1-k^n}{1-k}$.

If we have $10,000g \times \frac{moles}{235\, grams} \times \frac{6.022\times10^{23} atoms}{mole} = 2.56255 \times 10^{25}\, atoms$ of ^{235}U and if $k = 1.9$ then we have:

$$\frac{1-(1.9)^n}{1-1.9} = 2.56255 \times 10^{25}$$

$$(1.9)^n = 2.3063 \times 10^{25}$$

we can then take the natural logarithm of both sides to simplify

$$n = \frac{ln(2.3063 \times 10^{25})}{ln(1.9)} \approx 90.9869 \approx 91\, generations.$$

Prob. 6 solution by Baladitya

For each reaction, we have 1.9 neutrons produced in the reaction for each 1 neutron absorbed in the reaction. Those new neutrons will chain the fission for more U235.

The atomic mass of U235 is 235.04 amu, so there are 42.55 moles in 10 kg of U235.

Thus, for the k generations that it takes to fission 42.55 moles of U235,
$1.9^k = 42.55*(6.022*10^{23})$
$k = 91.14$
There requires 92 generations to fission 10kg of U235.

Problem #9.

First we calculate total energy output of a 1GW power plant:
1 GJ/sec x 31.6 x 10^6 sec/yr = 3 x 10^16J = 2 x 10^29 MeV

1 decay is about 200 Mev.

We assume that throughout the entire year, the reaction is kept at critical, so k ~ 1. This means we simply divide the total energy and energy per decay to obtain # of decays need: 2 x 10^26, which is about 1700 moles.

This corresponds to ~4 x 10^2 kg of pure U-235.

The density of Uranium is ~19g/cm^3, so we need ~0.020m^3 pure U-235.

The natural abundance of U-235 is .0072, so we divide the above numbers by 0.0072 to get the required unenriched Uranium volume & mass:

2.8 m^3 & 5.4 x 10^4 kg

Problem 10

Assume the Earth and Sun are spherically symmetric bodies.

a) Call u the dimensionless gravitational potential. Then:

$$u_{Earth} = -\frac{1}{c^2}\frac{GM_{Earth}}{r_{Earth}}$$

$$= -\frac{1}{\left(29979245800\,\frac{\mathrm{m}}{\mathrm{s}}\right)^2}\frac{\left(6.673\times10^{-11}\,\frac{\mathrm{m}^3}{\mathrm{kg}\cdot\mathrm{s}^2}\right)(5.9742\times10^{24}\ \mathrm{kg})}{6.3781\times10^{6}\ \mathrm{m}}$$

$$= -6.955\times10^{-14}$$

$$u_{Sun} = -\frac{1}{c^2}\frac{GM_{Sun}}{\mathrm{A.U.}}$$

$$= -\frac{1}{\left(29979245800\,\frac{\mathrm{m}}{\mathrm{s}}\right)^2}\frac{\left(6.673\times10^{-11}\,\frac{\mathrm{m}^3}{\mathrm{kg}\cdot\mathrm{s}^2}\right)(1.98892\times10^{30}\ \mathrm{kg})}{1.49598\times10^{11}\ \mathrm{m}}$$

$$= -9.871\times10^{-13}$$

b)

$$\frac{F_{Earth}}{F_{Sun}} = \frac{\left(\frac{GM_{Earth}m}{r_{Earth}^2}\right)}{\left(\frac{GM_{sun}m}{\mathrm{A.U.}^2}\right)}$$

$$= \left(\frac{M_{Earth}}{M_{Sun}}\right)\left(\frac{\mathrm{A.U.}}{r_{Earth}}\right)^2$$

$$= \left(\frac{5.9742\times10^{24}\ \mathrm{kg}}{1.98892\times10^{30}\ \mathrm{kg}}\right)\left(\frac{1.49598\times10^{11}\ \mathrm{m}}{6.3781\times10^{6}\ \mathrm{m}}\right)^2$$

$$= 1.514\times10^{3}$$

c) The gradients in the gravitational fields are given by the gradient operator (i.e. ∇). For central forces, $\nabla = \frac{\partial}{\partial r}$. Thus calling f the gradient, $f = \frac{\partial}{\partial r}\frac{GMm}{r^2} = -2\frac{GMm}{r^3}$. Therefore:

$$\frac{f_{Earth}}{f_{Sun}} = \frac{\left(-2\frac{GM_{Earth}m}{r_{Earth}^3}\right)}{\left(-2\frac{GM_{sun}m}{\mathrm{A.U.}^3}\right)}$$

$$= \left(\frac{M_{Earth}}{M_{Sun}}\right)\left(\frac{\mathrm{A.U.}}{r_{Earth}}\right)^3$$

$$= \left(\frac{5.9742\times10^{24}\ \mathrm{kg}}{1.98892\times10^{30}\ \mathrm{kg}}\right)\left(\frac{1.49598\times10^{11}\ \mathrm{m}}{6.3781\times10^{6}\ \mathrm{m}}\right)^3$$

$$= 3.551\times10^{7}$$

Thus the gradient from the Earth is bigger than the gradient from the Sun.
However, the gradient from the Sun (and also the moon) is important in determining the ocean tides, because the gradient from the Earth is spherically symmetric about the center of the Earth. On the other hand, the gradient from the Sun has a preferred direction, causing tides.

Problem 11

Because $1\ \mathrm{m}$ is much smaller than the radius of the Earth, we can approximate the gravitational force as uniform. Thus:

$$\begin{aligned} U &= mgh \\ &= (1.675 \times 10^{-27}\ \mathrm{kg})\left(9.81\,\frac{\mathrm{m}}{\mathrm{s}^2}\right)(1\ \mathrm{m}) \\ &= 1.643 \times 10^{-26}\ \mathrm{J} \\ &= 1.026 \times 10^{-7}\ \mathrm{eV} \end{aligned}$$

The kinetic energy for an ultracold neutron is at most $300\ \mathrm{neV} = 3 \times 10^{-7}\mathrm{eV}$, which is comparable to the gravitational potential energy from the Earth, so gravity needs to be accounted for in UCN experiments when dealing with the fastest ultracold neutrons.