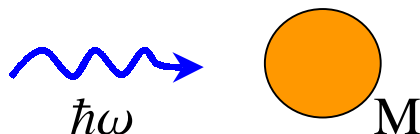


Laser Cooling: some very basic ideas

Let's say we have a two-level atom at rest:



If we bombard this atom with a resonant photon which is absorbed, the atom gets a kick in the direction of the original photon. The atom acquires momentum:

$$M \cdot v = \frac{\hbar\omega}{c}.$$

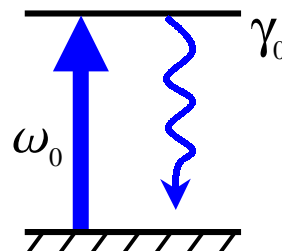
The corresponding energy is (for a sodium atom with mass number A=23):

$$E = \frac{p^2}{2M} = \frac{\hbar^2\omega^2}{2Mc^2} \sim \frac{(2 \text{ eV})^2}{2 \cdot 23 \cdot 10^9 \text{ eV}} \approx 10^{-10} \text{ eV}. \quad (1)$$

To appreciate how small this is, let us convert this energy into degrees Kelvin. Incidentally, this is called the single-photon recoil temperature limit (for obvious reasons):

$$T_{1\gamma} \approx 10^{-10} \text{ eV} \cdot 10^4 \frac{\text{K}}{\text{eV}} \approx 1 \mu\text{K} \quad (\text{for Na}). \quad (2)$$

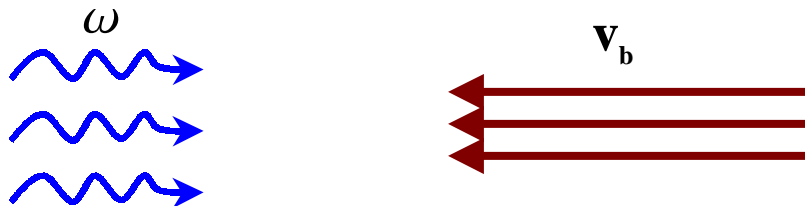
So far, we have only considered an absorption event. Now, an excited atom decays back to the ground state with rate $\gamma_0 = 1/\tau_0$ (this is not unlike some students who get excited about their work for a while, but then inevitably relax to their normal state).



An important point is that the fluorescence photon emitted in the process of de-excitation goes in a random direction (actually, sometimes this statement is not completely true, i.e. there are favored and disfavored emission directions, e.g. the familiar dipole radiation pattern, but this is not essential right now). When this happens, atom recoils in a random direction.

If we keep doing this: absorbing photons from a laser beam and emitting photons in random directions, we get the atom moving in the direction of the laser beam, and we also “heat” the motion in orthogonal directions as a result of “random walk” kicks.

What we’ve said so far is actually already enough to understand how people slow down and even stop atomic beams. For simplicity, let’s say we have an atomic beam where all atoms move with the same velocity \mathbf{v}_b .



We shine a monochromatic laser beam head-on onto the atoms. We need to tune the laser frequency in such a way that the photons are in resonance with the atomic transition (ω_0), i.e. we need to compensate for the Doppler shift:

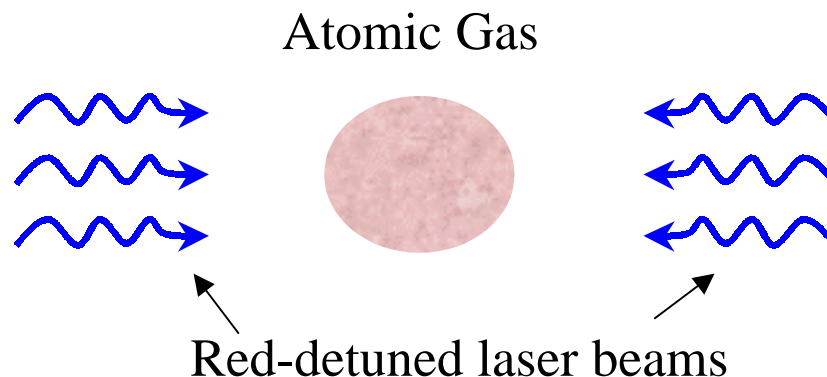
$$\omega_0 = \omega \cdot (1 + v_b/c) . \quad (3)$$

From (3) we see that the laser has to be “red-detuned”, i.e. its frequency has to be somewhat lower than the atomic resonance frequency ω_0 .

OK, so now the atoms happily interact with the photons and slow down somewhat. At this point, we have a slight problem since v_b has changed and the resonance condition (3) is no longer satisfied. If we want to continue slowing a certain group of atoms, we can chirp (gradually increase) the laser frequency. Another ingenious technique of keeping the atoms on resonance – Zeeman Slowing – is dealt with in the homework. In this technique, the atomic beam travels in a changing magnetic field, so the changing Zeeman shifts compensate for the slowing effect, keeping the atoms on resonance with a laser beam of a fixed frequency.

How does cooling work?

1-D



3-D: Multiply this by 3 (i.e. shine pairs of beams in three orthogonal directions).

This is called Optical Molasses. An interesting question is: how cold can you get the atoms using this method?

There appears to be no problem in cooling atoms to temperatures where the Doppler width becomes comparable with the natural width of the transition γ_0 . At this point, we have to be careful since the absorption probability loses its dependence on atomic velocity.

OK, so we have atoms that are sufficiently cold, so the absorption line has the width γ_0 , and we want to further cool the atoms. If we tune the laser so that $|\omega - \omega_0| \gg \gamma_0$, we will have very few laser-atom interactions, small cooling force. If we tune so that $\omega = \omega_0$, there is again no cooling force. Let's compromise and choose $\omega_0 - \omega = \gamma_0/2$ (remember, it is important that we are red detuned!).

Now our atoms get kicks both from left and from right, but if the atom is moving, say, to the right, it still gets more kicks from the right.

Say we have N absorption-spontaneous re-emission cycles per unit time. Since our atomic cloud is not moving as a whole, we can say that the average momentum $\langle \mathbf{p} \rangle$ imparted on the atoms is zero. On the other hand, we can say that $\langle p^2 \rangle$ is cooled by the mechanism we are discussing, but is also heated by random kicks from spontaneous emission, and also by the random nature of absorption events from left and right. We will determine the equilibrium value of $\langle p^2 \rangle$ from the balance of heating and cooling.

Let's introduce the characteristic velocity of cooled atoms:

$$\tilde{v} = \frac{\sqrt{\langle p^2 \rangle}}{M}. \quad (4)$$

Now we are ready to estimate the cooling rate. When a slowing photon is absorbed, we get a change in p^2 that can be estimated as:

$$(p - (\Delta p)_{i\gamma})^2 - p^2 \approx -2p(\Delta p)_{i\gamma} = -2M\tilde{v} \cdot \frac{\hbar\omega}{c}. \quad (5)$$

The fraction of "useful" cooling photons (i.e. the fraction of the kicks in the "right" direction out of the total number of kicks) can be estimated as $\frac{\omega \cdot \tilde{v} / c}{\gamma_0}$ (for $\omega \cdot \frac{\tilde{v}}{c} \ll \gamma_0$).

Finally, we write:

$$\left(\frac{\partial \langle p^2 \rangle}{\partial t} \right)_c \sim -N \cdot \frac{\omega \cdot \tilde{v} / c}{\gamma_0} \cdot 2M\tilde{v} \cdot \frac{\hbar\omega}{c} . \quad (6)$$

Now we deal with heating. Since this is a completely random process, here we neglect the cross-term $\sim p \cdot \Delta p$ in (5) and write (this is the random walk argument!):

$$\left(\frac{\partial \langle p^2 \rangle}{\partial t} \right)_h \sim N \cdot (\Delta p)_{1\gamma}^2 = N \cdot \frac{\hbar^2 \omega^2}{c^2} . \quad (7)$$

Finally, we have:

$$\left(\frac{\partial \langle p^2 \rangle}{\partial t} \right)_c + \left(\frac{\partial \langle p^2 \rangle}{\partial t} \right)_h = 0 , \quad (8)$$

from which we obtain:

$$M\tilde{v}^2 = \frac{\hbar\gamma_0}{2} .$$

Obviously, we were not careful about factors of two and such, but we got quite close to the exact result for the Doppler limit for temperature:

$$T_D = \frac{\hbar\gamma_0}{2} . \quad (9)$$

With $\gamma_0 \approx 2\pi \cdot 10$ MHz, we get:

$$T_D \approx \frac{1}{2} \cdot \frac{10^7 \text{ Hz}}{2 \cdot 10^{14} \text{ Hz} / eV} \cdot 10^4 \frac{K}{eV} \approx 200 \mu\text{K} . \quad (10)$$

Note that the Doppler limit T_D (9,10) and the one-photon recoil limit $T_{1\gamma} = \frac{\hbar^2 \omega^2}{2Mc^2}$ (1,2)

are two characteristic temperature scales in laser cooling. In fact, there is another characteristic temperature T^* which corresponds to $\Gamma_D \approx \gamma_0$ (where Γ_D is the Doppler width). It is an easy exercise to show that

$$T_D^2 = T_{1\gamma} \cdot T^* . \quad (11)$$

A few years ago, when people started experimentally approaching the Doppler limit, they were pleasantly surprised to find out that in certain cases they could cool atoms below the Doppler limit. How this can be (“Sisyphus” cooling, etc.) is well described in an October 1990 *Physics Today* article by C. Cohen-Tannoudji and W. Phillips. In fact, it turns out that it is also possible to do better than the recoil limit (“sub-recoil” cooling).

A final note: cooling does not mean trapping. A common technique nowadays is a combination of an optical molasses with a magnetic trap (Magneto-Optical Trap—MOT).