Optical Angular Momentum

Laguerre-Gaussian Modes of Laser Light

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Overview

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- Beth's 1936 experiment

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- Laguerre-Gaussian (LG_p^ℓ) modes

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- Production of LG_p^ℓ modes

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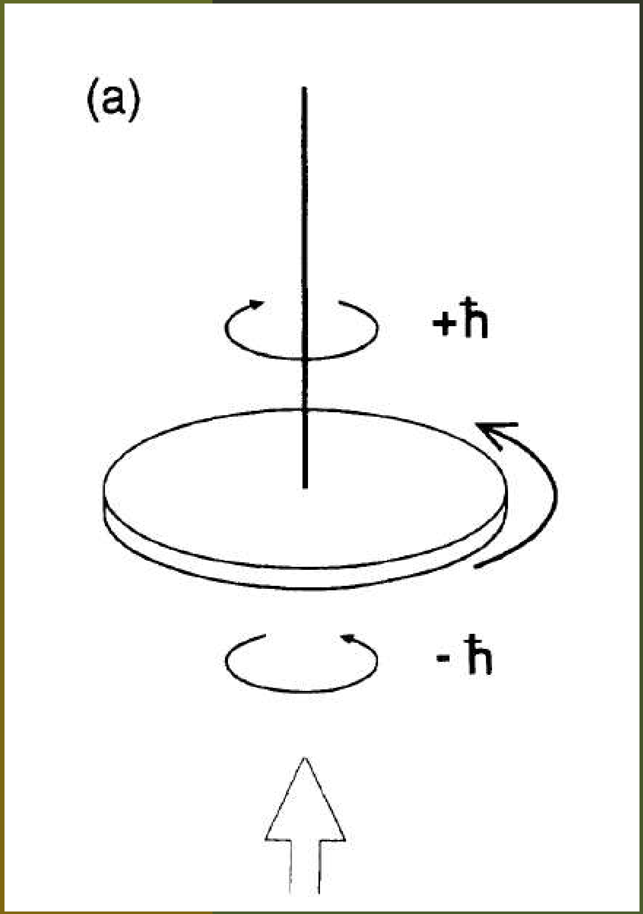
- Laguerre-Gaussian (LG_p^ℓ) modes
- Production of LG_p^ℓ modes
- Circularly polarized LG_p^ℓ beams

Beth's 1936 experiment



- Each photon in a $\sigma_\pm$ beam carries $\pm\hbar$ spin angular momentum
- Birefringent plate transforms σ_- beam to σ_+
- Plate experiences torque
- Both classical and quantum theories predict same value

R. A. Beth, Phys. Rev. **50**, 115 (1936)

Laguerre-Gaussian (LG_p^ℓ) modes

$$u_p^\ell(r, \phi) \propto r^\ell L_p^\ell(2r^2/w^2)\, e^{-r^2/w^2}\, e^{-i\ell\phi}$$

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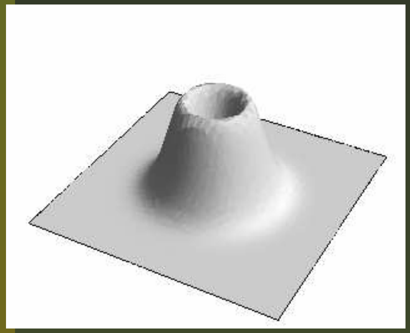
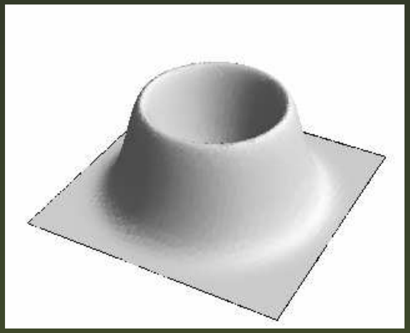
- Gaussian with beam waist w
- Vortex with *topological charge* ℓ
- Orbital angular momentum $\ell\hbar$ per photon
- Cylindrically symmetric solutions to the wave equation

LG_p^ℓ modes cont.

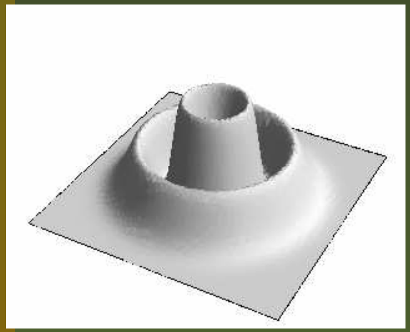
$|u_0^1|^2$



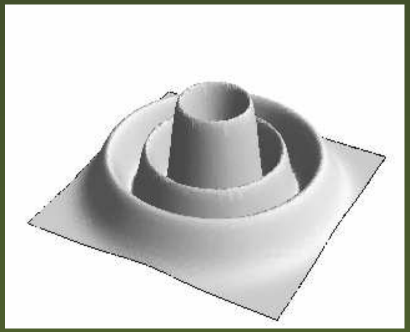
$|u_0^5|^2$



$|u_1^2|^2$



$|u_2^3|^2$



Production of LG_p^ℓ modes

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- Conversion from Hermite-Gaussian (HG_{nm}) modes

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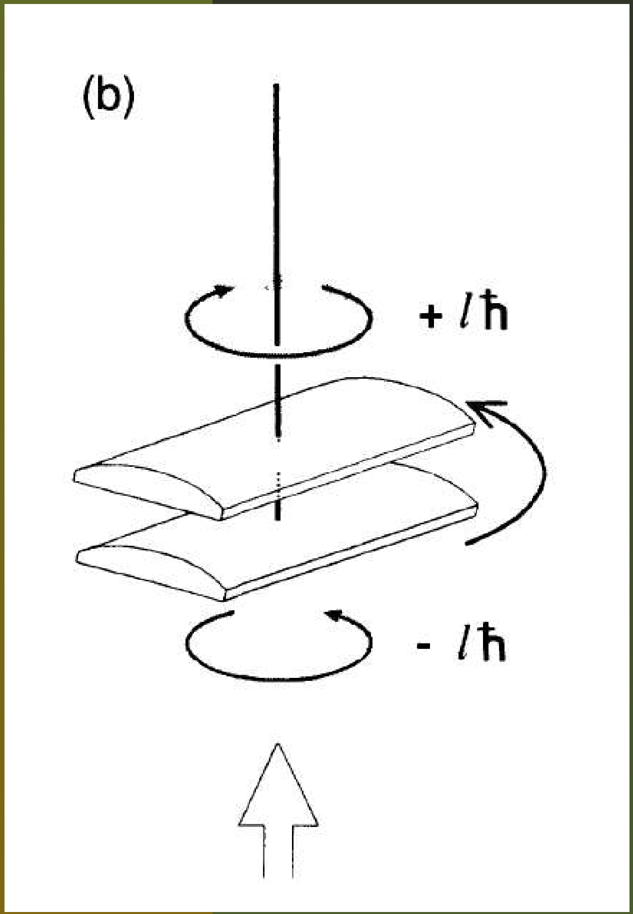
- Diffractive optics

Conversion of HG_{nm} to LG_p^ℓ modes

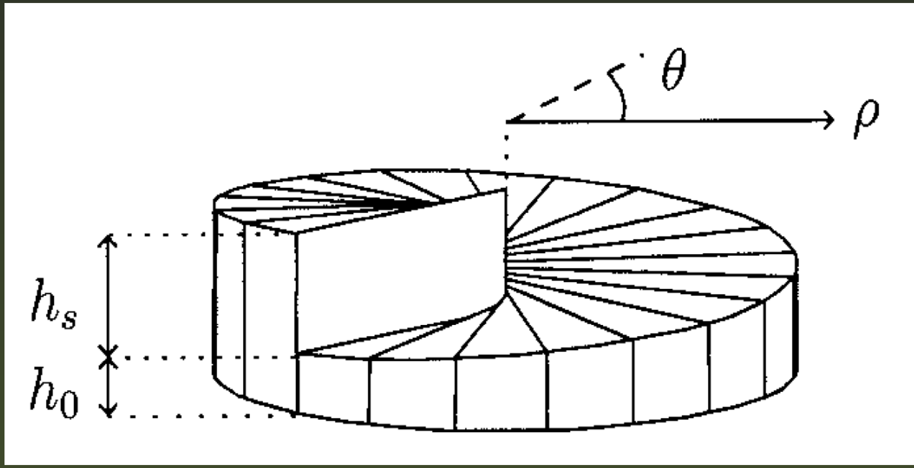


- Cylindrical lens converter: from HG_{nm} to LG_p^ℓ
- $\ell = m - n$ and $p = \min(m, n)$
- Introduces a Guoy phase shift on incident beam
- Combination of two lenses will experience torque
- For most lasers, high-order HG_{nm} modes are impractical

L. Allen *et al.*, Phys. Rev. A **45**, 8185 (1992)

J. Courtial and M.J. Padgett, Optics Comm. **159**, 13 (1999)

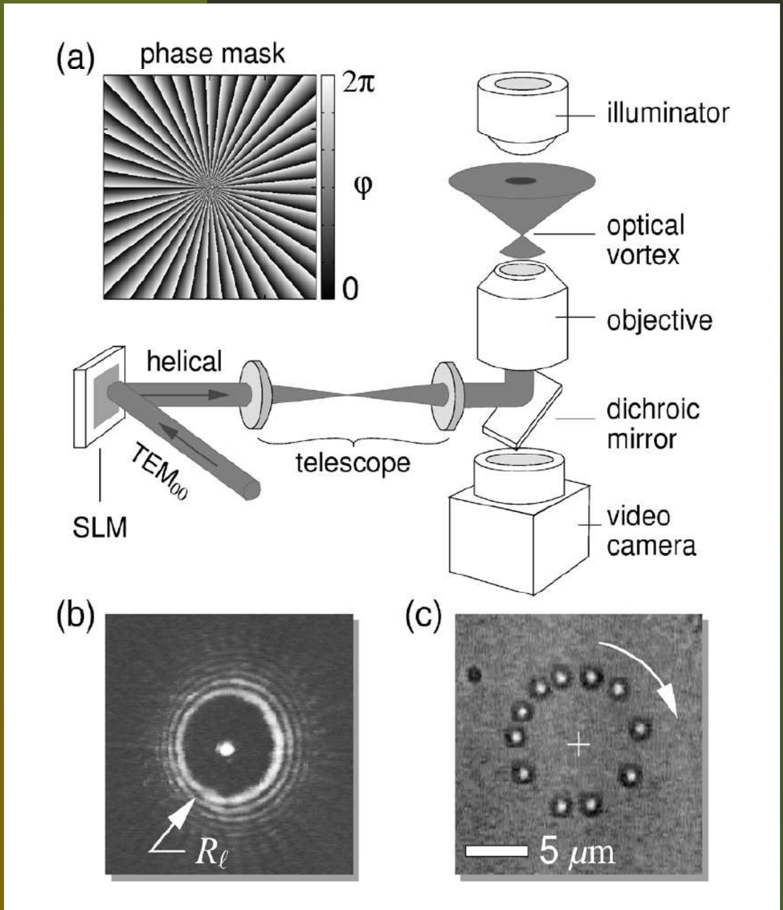
Spiral phase plates



- Azimuthally dependent phase delay
- Converts HG_{00} to LG_p^ℓ mode with charge $\ell = h_s(n - n_0)\lambda$
- Converts between any two LG_p^ℓ modes

G.A. Turnbull *et al.*, Optics Comm. **127**, 183 (1996)

Holographic gratings

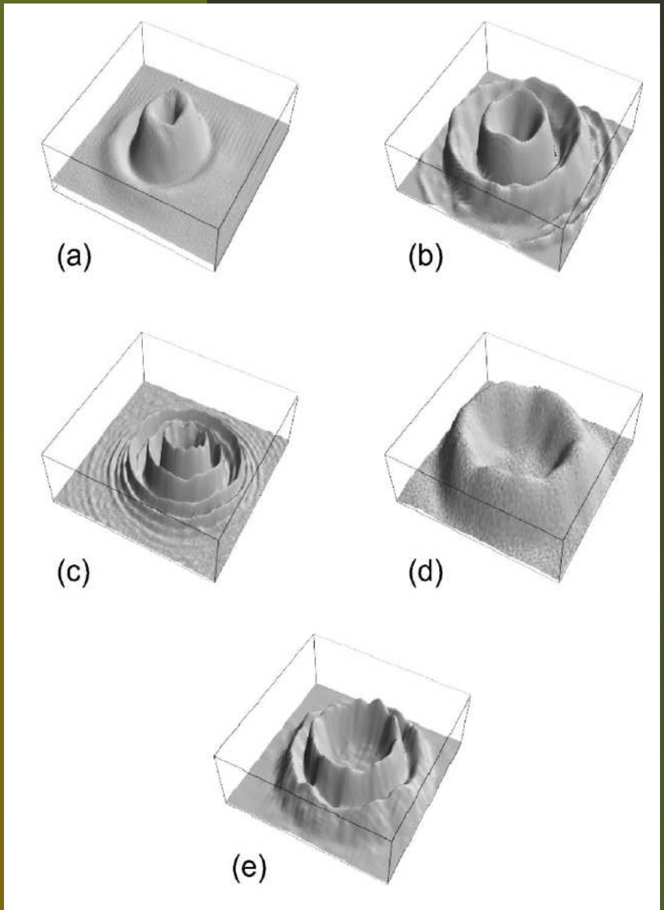


- Helical mode $\propto e^{i\ell\phi}$
- Topological charge $\ell = 40$
- Particle travels around circumference of vortex
- Radius of orbit $R_\ell \propto \ell$
- Photons contribute $\ell\hbar$ to angular momentum flux

$$T_\ell = R_\ell^3/(\ell \times \text{power}) \propto \ell^2/\text{power}$$

J.E. Curtis and D.G. Grier, Phys. Rev. Lett. **90**, 133901 (2003)

Diffractive optics

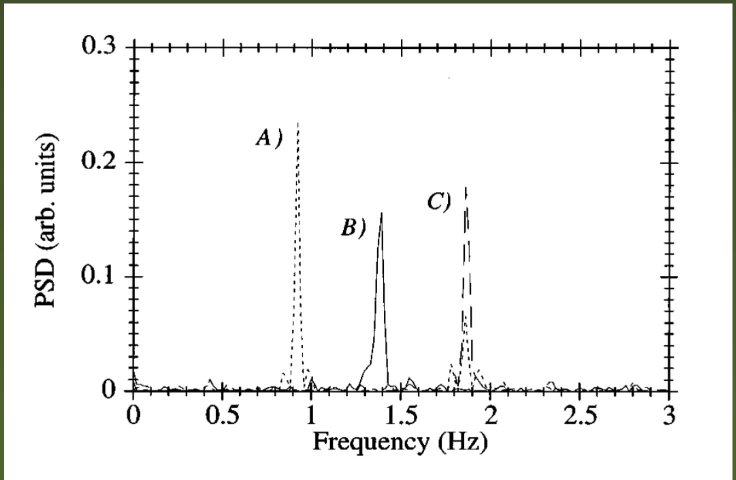


- Etch small scale structures on optical elements
- Use two diffractive optics to control intensity and phase
- Converts HG_{00} to arbitrary LG_p^ℓ mode
- High mode purity

S.A. Kennedy *et al.*, Phys. Rev. A **66**, 043801 (2002)

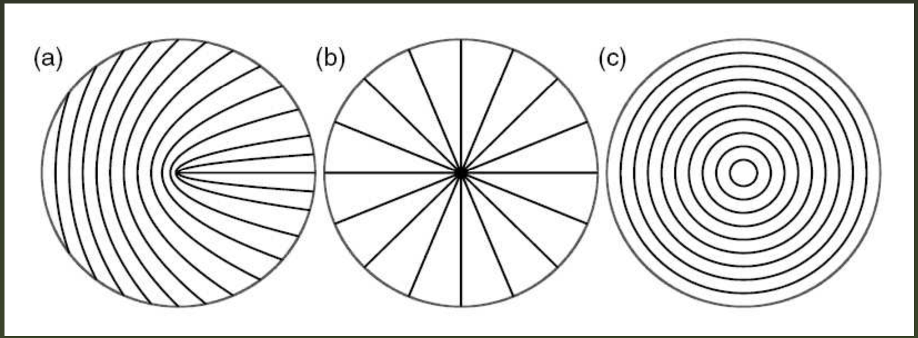
Circularly polarized LG_p^ℓ modes

- Shine a He-Ne laser on CuO particles in a viscous fluid
- Phase hologram produces LG_3^0 mode
- Particles experience torque $\tau \propto \ell + \sigma_z$
- ... and a drag torque proportional to angular velocity
- Measure equilibrium angular velocity for $\sigma_z =$ -1, 0, +1



M.E. Friese *et al.*, Phys. Rev. A **54**, 1593 (1996)

Spin-to-orbital conversion



- Anisotropic media transfer spin angular momentum to matter (e.g., birefringent media and liquid crystals)
- Inhomogeneous isotropic media transfer orbital angular momentum
- "q-plates" are both anisotropic and inhomogeneous
- Optical axis orientation specified by $\alpha(r, \phi) = q\phi + \alpha_0$

L. Marrucci *et al.*, Phys. Rev. Lett. **96**, 163905 (2006)

Spin-to-orbital cont.

Jones matrix for the q-plate:

$$\mathbf{M} = \mathcal{R}(-\alpha) \cdot \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \mathcal{R}(\alpha) = \begin{pmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{pmatrix}$$

Spin-to-orbital cont.

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Incoming is left-circularly polarized plane wave.

Outgoing wave is right-circularly polarized with charge $\ell = 2q$:

$$\mathbf{E}_{\text{out}} = \mathbf{M} \cdot \begin{bmatrix} 1 \\ i \end{bmatrix} \propto e^{i2q\phi} \begin{bmatrix} 1 \\ -i \end{bmatrix}$$

Spin-to-orbital cont.

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Each photon exchanges $\pm 2\hbar(q-1)$ angular momentum with plate.

Summary

Circularly polarized light has $\sigma_z \hbar$ angular momentum per photon

Laguerre-Gaussian modes have $\ell \hbar$ angular momentum per photon

Beams with angular momentum can make small things rotate

Fancy materials can convert spin to orbital angular momentum