Due: Thursday, 02/03

1. This problem gives exercises in atomic units ($\hbar=m_e=e=1$, $c=1/\alpha\approx137$).
a) What is the Bohr magneton in atomic units?
b) What is the atomic unit of electric field in Volts/cm?
c) What is the atomic unit of magnetic field in Gauss?
d) What are the ground state binding energies of positronium, muonium, atomic deuterium, singly ionized helium-four, in atomic units?

2. Elliptically polarized light propagating along z can be thought of as superposition of two linearly polarized waves with electric fields along x and y, respectively, which are at the same frequency, but generally, have different amplitudes and a phase difference :

$$\begin{aligned} E_x &= E_x^0 \cos\omega t, \\ E_y &= E_y^0 \cos(\omega t + \varphi) \end{aligned}$$

a) Find the angles between the x axis and the principal axes of the polarization ellipse.
b) Ellipticity is defined as the arctangent of the ratio of the polarization ellipse's semi-axes. What is the ellipticity for linearly polarized light, left circularly polarized light, and right circularly polarized light?

3. Research project on angular momentum of light. In the quantum picture, a photon of circularly polarized light carries intrinsic angular momentum with projection +1 or -1 ($\hbar$) on the propagation direction. Presumably, one should be able to derive this result starting from considering a classical plane-wave electromagnetic field. There appears to be a problem with this. Indeed, in order to have angular momentum along z, there must be linear momentum in the system in the x-y plane. However, since the momentum density of electromagnetic field is proportional to $\boldsymbol{E}\times\boldsymbol{B}$, and both electric and magnetic fields in the plane wave are transverse, it appears there is no transverse momentum in the system. The assignment is to find (either by yourself and/or in the literature) and report a resolution to this "paradox." The report should be presented in the form of a "paper" which should be as short as possible, but should contain the usual elements: an Introduction formulating the problem, a clearly stated Conclusion, and References. Assume that the reader is an intelligent and curious person like you, who is, however, not yet familiar with the problem.