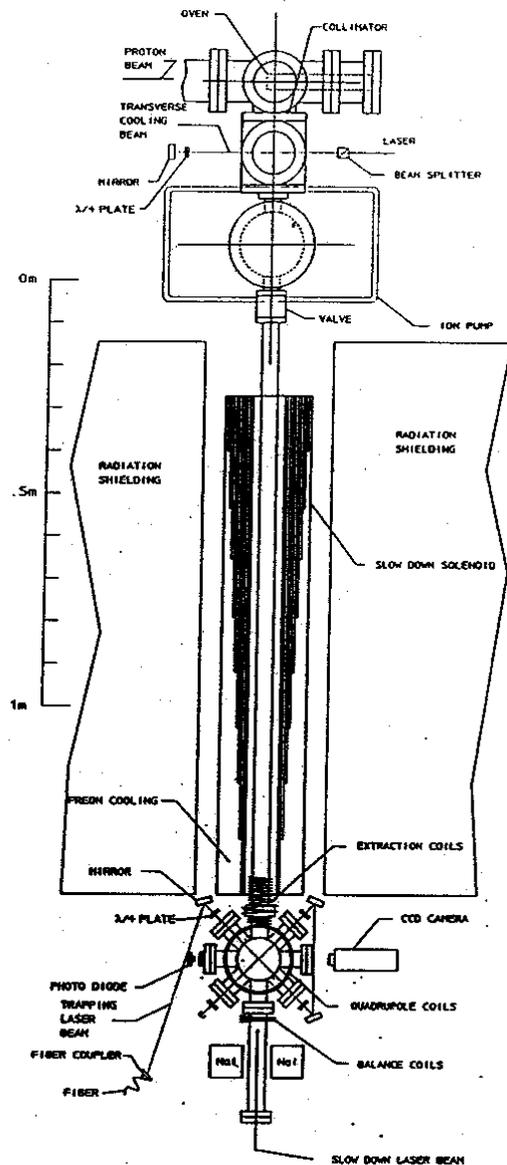


5. Consider the motion of a single electron (or positron) in geonium. A) Show that the correction to the axial oscillation frequency due to the spin orientation, cyclotron motion and magnetron motion is given by:  $\delta \omega_z \approx [m+n+1/2+(\omega_s/\omega_c)q]\delta$ ,  $\delta=2\mu_B\beta/m_e \omega_z$ , where  $m$  is the spin magnetic quantum number,  $n$  and  $q$  are the cyclotron and magnetron quantum numbers,  $\omega_s$ ,  $\omega_c$ ,  $\omega_z$  are the spin, cyclotron, and axial frequencies, and  $\beta$  is the parameter of the inhomogeneous magnetic field; see [H. Dehmelt, Am. J. Phys. 58, 17-27 \(1990\)](#). B) Estimate the dimensions of the electron wavepacket for  $m=n=q=0$ .

6. In an ongoing experiment at the Lawrence Berkeley Laboratory, the short-lived radioactive  $^{21}\text{Na}$  atoms (22.5 sec half-life) are produced by bombarding stable magnesium atoms with a beam of protons in the reaction  $^{24}\text{Mg}(p, \alpha)^{21}\text{Na}$ . The resulting sodium atoms are evaporated and a fraction of them is subsequently cooled and trapped in a magneto-optical trap. A schematic of the apparatus is shown in the figure. The magnesium atoms are placed in an oven where the interaction with a proton beam from an accelerator occurs. When the oven is heated (to  $500^\circ\text{C}$ ), an atomic beam of  $^{21}\text{Na}$  is produced. As the atoms in the beam travel toward the magneto-optical trap, their transverse velocities are first reduced with transverse laser beams in the transverse-cooling region. The atoms then travel a distance  $L=1.2\text{ m}$  in a region where they are



slowed to essentially zero longitudinal velocity by interaction with a counter-propagating laser beam before the actual trapping occurs. It is this slow-down region we will be concerned with in this problem.

The slowing laser beam is tuned near the  $F=2 \rightarrow F'=3$  hyperfine component of the  $3^2S_{1/2} \rightarrow 3^2P_{3/2}$  transition (D<sub>2</sub>-line,  $\lambda=589$  nm, excited state lifetime  $\tau=16$  ns) and has  $\sigma^+$  polarization. The resonance frequency of the atoms in the beam is Doppler-shifted. As the atoms slow down, the Doppler shift decreases. In order to keep the atoms in resonance with the laser beam used for slowing, a spatially varying magnetic field is produced with a solenoid with a non-uniform winding.

- a) Assuming that the atoms are always in resonance with the slowing laser light, estimate the time it takes the atoms to stop for the case of the saturation parameter  $\kappa=1$ . Compare this to the stopping time in the present apparatus. From this comparison, estimate the actual saturation parameter.
  - b) Assume for simplicity that all atoms initially have the same longitudinal velocity corresponding to the oven temperature and neglect the hyperfine splitting. (Both of these assumptions are not realistic but they simplify the calculations immensely and give the correct estimates of orders of magnitude of the parameters involved.) Calculate the size and longitudinal coordinate dependence of the magnetic field produced by the slow-down solenoid. Note: the laser is tuned near the zero-field resonance frequency.
7. Suppose we have a volume containing gas of molecules of mass  $M$  at density  $n(\text{cm}^{-3})$  and temperature  $T$ . The molecules have collisional cross-section  $\sigma$ . What is the mean free path of a molecule between collisions and the average time between collisions? Now suppose we add some gas of another sort (buffer gas) into the volume. What will be the effect of the buffer gas on the rate of collisions of the original molecules between themselves?