Why Can Bose-Einstein Condensates Interfere?

Figure ruthlessly stolen from W. Ketterle’s Web Page at http://rleweb.mit.edu/rlestaff/p-kett.htm
Outline of Talk

- Canonical relation between phase and number
- A brief historical tangent
- What all the fuss was about
- The theoretical solution
- The experimental proof
- The end
Consider a collection of bosons in a harmonic oscillator potential in a mode with frequency $\omega$

$$E = n \hbar \omega$$

$\Delta E = \Delta n \hbar \omega$

*Heisenberg Says:*

$$\Delta E \Delta t = \Delta n \hbar \omega \Delta t = \Delta n \hbar \Delta \phi \geq \hbar$$

$$\Delta n \Delta \phi \geq 1$$
A brief historical tangent

\[ N = a^+ a \]

Dirac suggested in 1927 a possible "phase operator:"

\[ e^{i \phi} = a \hat{N}^{-1/2} \]

So that

\[ [\hat{N}, \hat{\phi}] = i \]

\[ \Delta n \Delta \phi \geq \frac{1}{2} \]

In 1962, Peter Carruthers assigned the analysis of Dirac’s phase operator as a problem in an advanced quantum mechanics class in order to get his students to do the work for him. He offered a "bonus" that turned out to be a single Budweiser Beer.

What all the fuss was about?

- "Problem" is than in a BEC, you can in principle determine without doubt how many atoms you have. That would mean that the phase of a BEC is meaningless.

- Same "problem" plagued ultra-low people.

- Kagen et. al. predicted that at first BEC would be a "semi-condensate" with long-range order (phase) occurring on a longer time scale.

  Bose-Einstein Condensation, ed. A Griffin et. al., 1995, p. 202

- Stoof predicted that a coherent BEC would form immediately.

  Bose-Einstein Condensation, ed. A Griffin et. al., 1995, p. 226
S. M. Barnett offered a solution:

“Our simple idea can be summed up in one sentence. A Bose-Einstein condensate has a preferred phase because it is meaningful, on a macroscopic timescale, to ascribe one to it.”


- Since the BEC is a quantum system, it’s properties depend “in a large part” on the measurements you can perform on it.

- The BEC is an open system. You can gain/lose atoms

- Therefore BEC are note made in pure states. BEC is highly entangled with environment. Use a density state to describe mixed state (typically diagonal in Fock state representation).
What do they do from there?

- They describe in more detail the nature and representation of a BEC from both a Fock state representation and a coherent state representation. They conclude that both are legitimate.

- They then discuss how the density matrix for a damped harmonic oscillator will evolve.

- They then show that number states and coherent states have decay rates given by:
  - \(2n \Gamma\) – rate of changing number state (live until one particle is lost)
  - \(<n>^{1/2} \Gamma\) – rate of leaving a coherent state (can live much longer)
  - \(\Gamma\) – rate of decay of mean field

- They then find that the spread in phase is like \(\frac{1}{2} <n>^{-1/2}\)

The experimental proof