Nonlinear optics with single quanta

A brief excursion into cavity quantum electrodynamics

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Physics 208

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What characterizes linear systems?

1. Start with input X
2. X impinges upon *stuff*
3. Output Y observed
4. *Stuff* can be characterized by transfer function T, where T = Y/X
5. Transfer function T is a very useful tool (esp. for systems engineers) when *stuff* is not modified by input parameter X

What makes a system *nonlinear*?

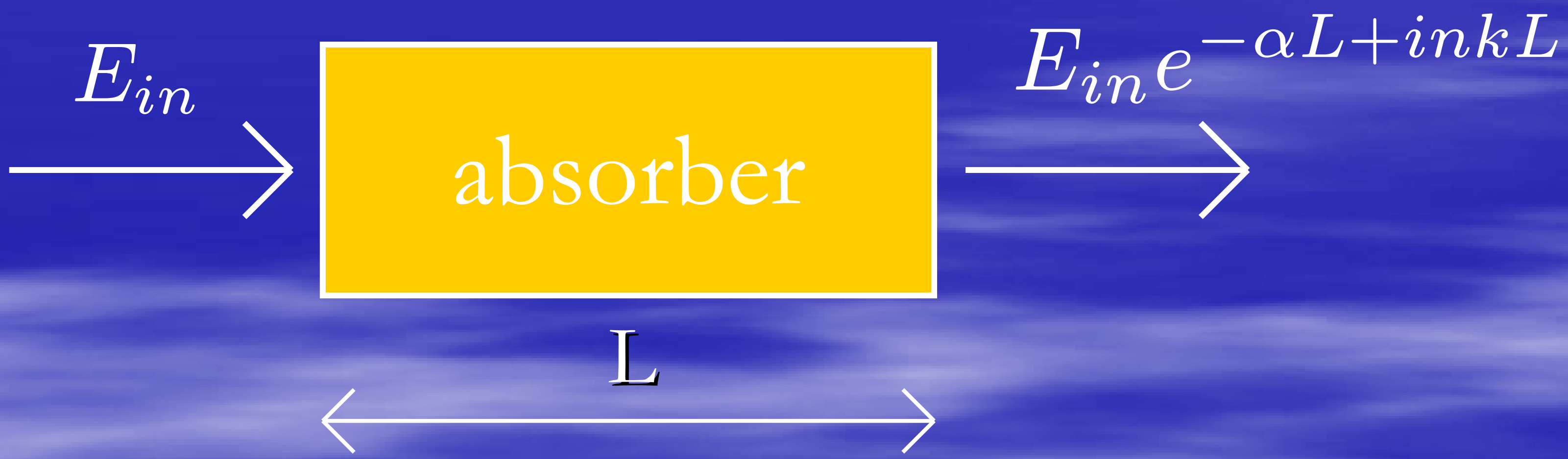
1. Start with input X impinging upon *nonlinear stuff*
2. Transfer function T becomes dependent on X (stuff modified by X)
3. Output Y = T(X)*X
4. Could still characterize *stuff* with transfer function T = Y/X, but usefulness is greatly diminished by the fact that T is now an explicit function of X parameters

Nonlinear optical systems (to-date)

- Macroscopic $\chi^{(2)}$ and $\chi^{(3)}$ systems have been the focus of the class thus far
- Some of the familiar nonlinear phenomena we've discussed are:
  - Sum- and difference-frequency generation
  - Intensity dependent refractive index
  - Nonlinear optical rotation
  - etc.
- To really probe these phenomena effectively, a large input intensity is a plus, if not a *must*

Saturation - Nonlinear optics on the cheap

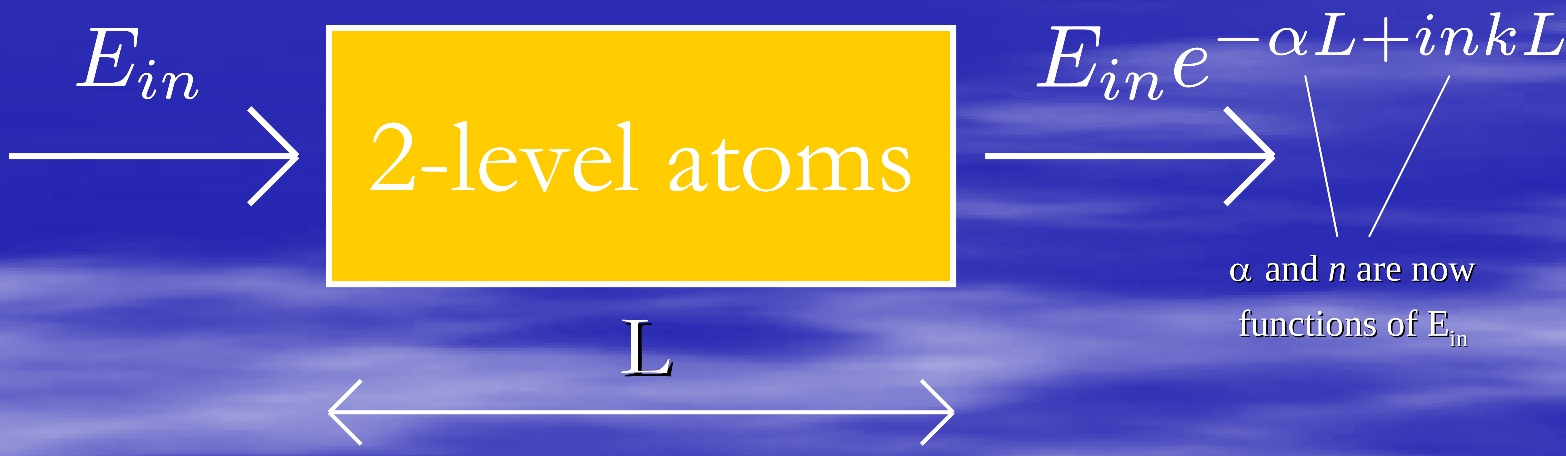
- Normal absorption



- Still linear system: $T = \frac{E_{out}}{E_{in}} = e^{-\alpha L}e^{inkL}$

Saturation - Nonlinear optics on the cheap

- What if absorber is ensemble of two-level atoms?
  - Ensemble can <u>saturate</u> if intensity is large
  - Real and complex refractive indices become functions of input intensity



- No longer linear system as T depends on E_{in}
- In particular, $\alpha = \frac{\alpha_o}{1+\frac{I}{I_s}}$

Saturation - Nonlinear optics on the cheap

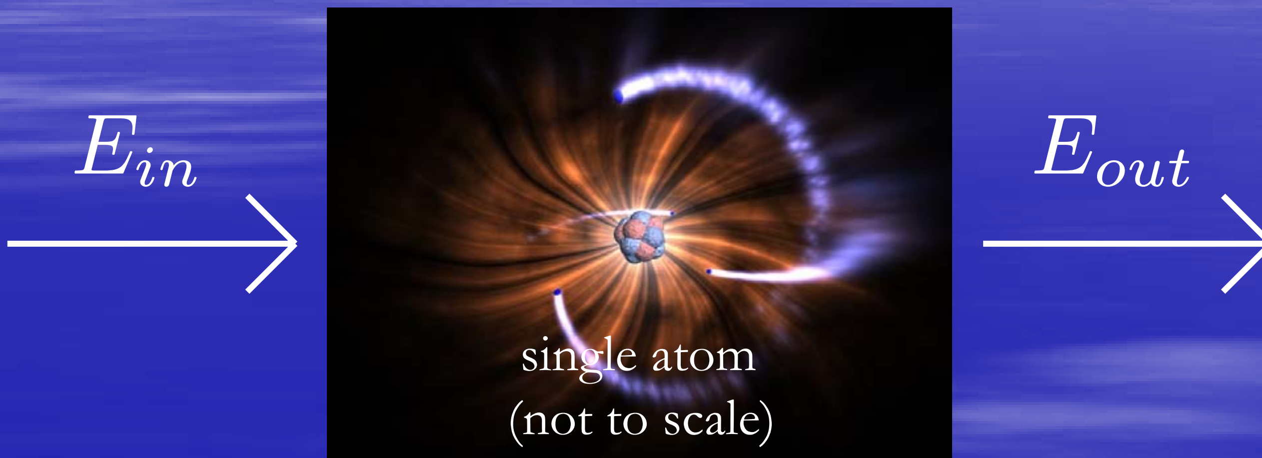
- In what sense is saturation *cheap*?
  - Saturation intensity can be quite small, ~10 mW/cm^2 for room temp gas
  - Easily within reach of cheap diode lasers (< $100)
- How does this relate to nonlinear optics as we've discussed it so far?

$$P(t) = \chi^{(1)}E(t) + \chi^{(2)}E(t)^2 + \chi^{(3)}E(t)^3 + \ldots$$

$$\alpha = \frac{\alpha_o}{1+\frac{I}{I_s}} = \alpha_o[1 - (\frac{I}{I_s}) + (\frac{I}{I_s})^2 - (\frac{I}{I_s})^3 \ldots]$$

What happens if system is *not* macroscopic?

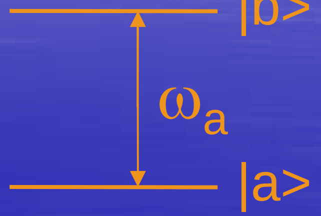
What happens if system is *not* macroscopic?

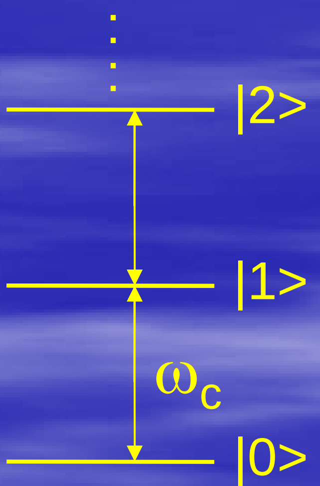


- Immediately nonlinear (saturable)
- Can even get other nonlinear-like behavior out
  - wave mixing (Raman scattering)
  - multi-photon processes can yield harmonic generation
- However, hard to observe single atom effects

Nonlinear optics with single quanta (atoms and photons)

- Ingredients:
  - two-level atoms
  - discrete light quanta
  - interactions





$$\mathcal{H} = \omega_a \sigma^+ \sigma^- + \omega_c a^+ a - \boldsymbol{d} \cdot \mathcal{E}$$

$$\boldsymbol{d} = \boldsymbol{d}_o(\sigma^+ + \sigma^-)$$

$$\mathcal{E} = \mathcal{E}_o(a^+ + a)$$

Fine for strong field ($\langle a^+ a \rangle$ large), but can we get a *single* atom to interact strongly with a *single* photon?

A single atom in the grips of a single photon

(or how I learned to stop worrying and make $d \cdot \mathcal{E}$ large)

State space: { |a,0> , |a,1> , |b,0> , ~~|b,1>~~ }

$$-\mathbf{d} \cdot \mathcal{E} = g_o(\sigma^+ a^- + \sigma^- a^+)$$

$$\mathcal{H} = \omega_a \sigma^+ \sigma^- + \omega_c a^+ a - \mathbf{d} \cdot \mathcal{E}$$

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$$\boldsymbol{d} = \boldsymbol{d}_o(\sigma^+ + \sigma^-)$$

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$$\begin{pmatrix} \omega_a & g_o \\ g_o & \omega_c \end{pmatrix}$$

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Single atom/photon Hamiltonian →
(low excitation regime)

$$\begin{pmatrix} \omega_a & g_o \\ g_o & \omega_c \end{pmatrix}$$

How big is g_o?

Zero point energy of photon → $\frac{\hbar\omega}{2} = \epsilon_o E^2 V$

Electric field of an empty photon mode → $E = \sqrt{\frac{\hbar\omega_c}{2\epsilon_o V}}$

Therefore,

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Therefore, $g_o = -d_o \cdot E = -d_o \sqrt{\frac{\hbar\omega_c}{2\epsilon_o V}} \sim d_o \sqrt{\frac{\omega_c}{V}}$

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Single atom/photon Hamiltonian →
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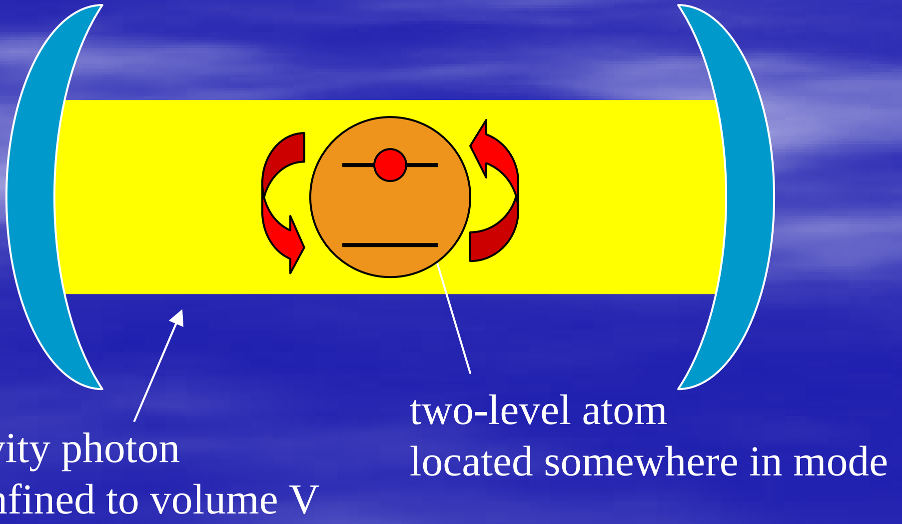
$$\begin{pmatrix} \omega_a & g_o \\ g_o & \omega_c \end{pmatrix}$$

Cartoon picture:



A single atom in the grips of a single photon

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How does one *embiggen* this quantity? ➡ $$d_o \sqrt{\frac{\omega_c}{V}}$$

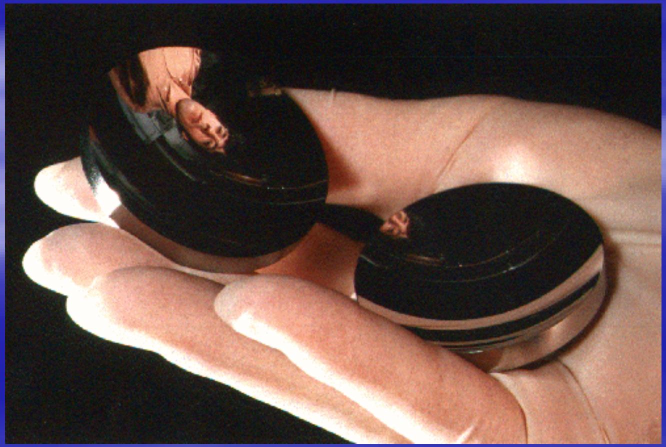
Finite V a <u>must</u>… the smaller the better!

Increase dipole moment

$$d_o \sim n^2 e\, a_o$$

(though $\omega \sim n^{-3}$)

Rydberg atoms (n big) + microwave cavities



Serge Haroche at ENS (France)

Shrink volume & keep ω_c large

Volume ~ 10^4 μm^3

use *optical* (or NIR) photons

Jeff Kimble at Caltech

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State space: { |a,0> , |a,1> , |b,0> , ~~|b,1>~~ }

$$\begin{pmatrix} \omega_a & g_o \\ g_o & \omega_c \end{pmatrix}$$

$$\Omega_\pm = \frac{\omega_a+\omega_c}{2} \pm \sqrt{\left(\frac{\omega_a-\omega_c}{2}\right)^2 + g_o^2}$$

if $\omega_a = \omega_c = \omega_o$ and $|\Psi(0)\rangle = |b,0\rangle$, then $|\Psi(t)\rangle = cos(g_o t)e^{-i\omega_o t}|b,0\rangle + sin(g_o t)e^{-i\omega_o t}|a,1\rangle$

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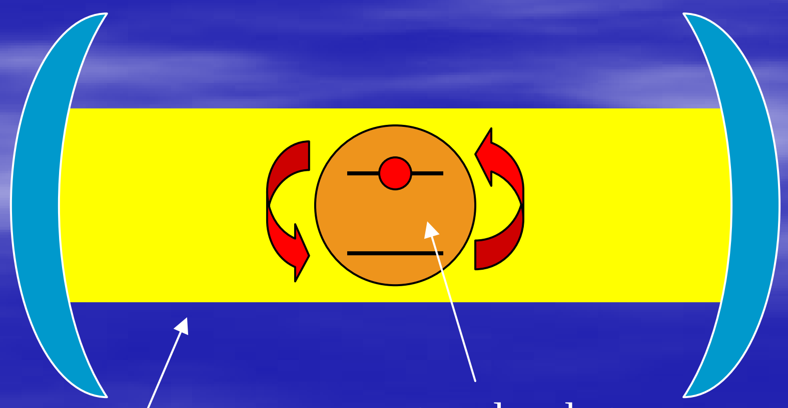
Single atom/photon Hamiltonian →
(low excitation regime)

$$\begin{pmatrix} \omega_a & g_o \\ g_o & \omega_c \end{pmatrix}$$

We've overlooked something…

Cartoon picture:



A single atom in the grips of a single photon

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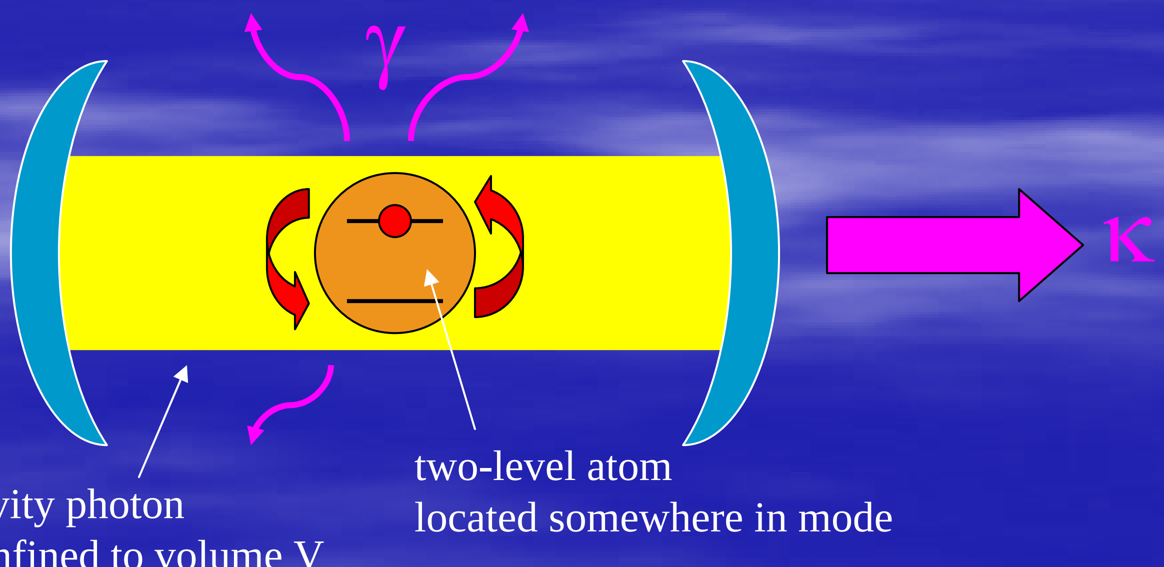
Single atom/photon Hamiltonian → $\begin{pmatrix} \omega_a - \frac{i\gamma}{2} & g_o \\ g_o & \omega_c - \frac{i\kappa}{2} \end{pmatrix}$

(low excitation regime)

We've overlooked something…decay!

Cartoon picture:



A single atom in the grips of a single photon

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State space: { $|a,0\rangle$, $|a,1\rangle$, $|b,0\rangle$, ~~$|b,1\rangle$~~ }

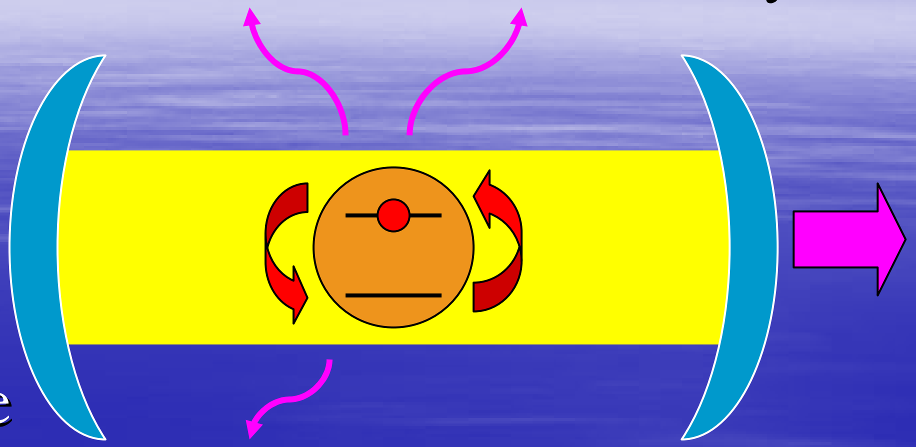
$$\begin{pmatrix} \omega_a - \frac{i\gamma}{2} & g_o \\ g_o & \omega_c - \frac{i\kappa}{2} \end{pmatrix}$$

$$\Omega_\pm = \frac{\omega_a + \omega_c}{2} - \frac{i\gamma + i\kappa}{4} \pm \sqrt{\left(\frac{\omega_a - \omega_c}{2} - \frac{i\gamma - i\kappa}{4}\right)^2 + g_o^2}$$

Sounds easy… what's the catch?

- Controlling decay rates
  - Shiny, shiny mirrors
  - Super-polished surfaces
  - Tiny mode volumes to increase g over κ, γ
- Controlling the atom
  - Atom are wily… must get them in cavity mode and "keep them there"
  - Precise control of atom position is trending towards the realm of cooling/trapping and other complicated schemes
- Low light detection
  - Must deal with low photon number states

What has cavity QED done for me lately?

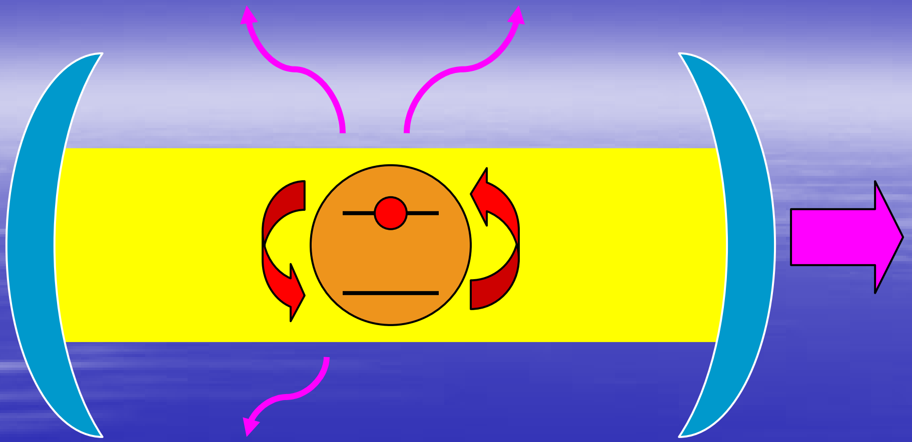


- Optical bistability
- Vacuum Rabi splitting
- Quantum phase gates
- Atomic motion microscope
- Distant atom-photon and atom-atom entanglement
- State-insensitive strongly coupled CQED
- Single-photon generation
- Discrete atom number counting
- "Single-atom laser"
- Solid-state implementation

In the near future…

- Deterministic Raman CQED
- Fock state generation of γ's
- Coupled cavities
- Fully operation quantum computer capable of factoring any number into its constituent primes, thereby rendering all modern cryptographic systems useless and allowing John Ashcroft's successor to read your email.. *

* don't hold your breath, unless you want to fund us in which case we'll have it ready next year



References

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2. Turchette, Q. *Ph.D. thesis*. Caltech (1997)
3. Boyd, R. W. *Nonlinear Optics*. Academic Press (2003)
4. Turchette, Q.A. *et al.* PRL **75**, 4710 (1995)
5. Jeff Kimble and Serge Haroche's websites
6. www.google.com (Why didn't I buy that stock?!?! It even translates French!)