

Due: Wednesday, 11/03/2004

The following two problems were inspired by Yongmin Liu's presentation on left-handed materials and artificial magnetic resonators.

19. When we discuss electric-dipole interactions of atoms with light, we often use the "electron on a spring" model. Devise a similar classical model for magnetic-dipole interactions.

An interesting related issue is the correspondence of the classical and quantum pictures. Let us discuss this in the context of the electron on a spring model. Classically, in order to have electric-dipole radiation, we need an oscillating (or rotating) dipole. Now, an atom in a given quantum state does not at all represent an oscillating dipole. In order to produce an oscillating dipole, we need to have a coherent superposition of non-degenerate states of opposite parity (you can see this by considering a superposition of an S and a P orbital, for example). By the way, this argument also shows that there is no classical picture for spontaneous emission from an excited state – classically, such states should be stationary (as they do not correspond to an oscillating and thus radiating dipole). Give a corresponding discussion for magnetic-dipole radiation.

20. "Left-handed materials" is a (unfortunate, as it has absolutely nothing to do with chirality normally associated with handedness) term describing a system where the electric and magnetic permittivities ϵ and μ are both negative. For an electromagnetic wave propagating in such a medium, the wave vector (\mathbf{k}) is in the direction opposite to $\mathbf{E} \times \mathbf{B}$, so, for example, one needs to use the left-hand instead of the right-hand rule to find the direction of \mathbf{B} from the directions of \mathbf{k} and \mathbf{E} .

Now to the (open-ended) problem. Attempt to design a left-handed material out of an atomic vapor. First see what you would want in terms of the energy levels and transitions in the atom, and then, we can see if, indeed, such a system can be found in practice. Estimate the required density of the atoms. Make sure the system is reasonably transparent.

If you succeed, this would be a great way to construct left-handed materials for optical range!