Due: Wednesday, 10/06/2004

12. Consider sum-frequency generation by two plane waves ($\omega_{1,2}$) falling normally on a lossless nonlinear medium.

a). Explain why the coupled amplitude equations can be written in the form ($\omega_3=\omega_1+\omega_2$)

$$\frac{dA_3}{dz}=\frac{8\pi i d\omega_3^2}{k_3c^2}A_1A_2e^{i\Delta kz}, \quad \frac{dA_{1,2}}{dz}=\frac{8\pi i d\omega_{1,2}^2}{k_{1,2}c^2}A_3A_{2,1}^*e^{-i\Delta kz},$$

with one and the same effective nonlinear coefficient d. What is the physical origin of the symmetry responsible for this? (See Boyd's book.)

b). Using these equations, show that

$$\frac{d}{dz}\left(\frac{I_1}{\omega_1}\right)=\frac{d}{dz}\left(\frac{I_2}{\omega_2}\right)=-\frac{d}{dz}\left(\frac{I_3}{\omega_3}\right) \qquad \text{(Manley-Rowe relations)},$$

where I_i is the intensity at ω_I. What is the physical meaning of these relations?

13. Obtain characteristic values of conversion efficiency into the second harmonic for a focused visible input beam of a). a cw laser, P=100 mW; b). a pulsed laser, Q=10 mJ in a 10 ns pulse, in a nonlinear crystal with and without phase matching.

14. Based on data from literature on the values of refractive indices n_o, n_e of ADP ($NH_4H_2PO_4$) crystals, calculate the type I phase-matching angle for frequency doubling of Nd-YAG laser light (λ=1.06 μ) at room temperature.

15. What is a characteristic value of the angle between the normal to the wavefront (phase velocity) and the Poynting vector (group velocity) for an extraordinary wave in a uniaxial crystal?