Due: Wednesday, 9/22/2004

6. Self-focusing. A beam of a Q-switched Nd-YAG laser ($\lambda = 1.06\ \mu$) has a diameter of 1 mm. The duration of a pulse is 7 ns, and the pulse energy is 100 mJ. The beam propagates through a slab of glass (e.g., fused silica) with a length of $L = 1$ cm. Due to the self-focusing effect, the glass slab acts as a lens.

a) What is the principal physical mechanism leading to the lensing effect?
b) Is the resulting lens a focusing or a defocusing lens?
c) Estimate the focal length of the lens.
d) Discuss the scaling of the focal length with the beam diameter, pulse energy, and pulse duration.

7.

8.3 Spectrum of frequency-modulated light

Consider a field $\mathcal{E}(t)$, oscillating at central frequency ω_0, which is modulated with frequency Ω with modulation depth $m\Omega$ (m is the *modulation index*).

$$\mathcal{E}(t) = \mathcal{E}_0 \exp\left[i\omega_0 t + im \sin \Omega t\right] . \tag{8.19}$$

Using the standard Bessel function identity [see, for example, Siegman (1986), Section 27.7)]

$$e^{im \sin \Omega t} = \sum_{k=-\infty}^{\infty} J_k(m) e^{ik\Omega t} , \tag{8.20}$$

the spectrum of the field can be represented as a sum of frequency components (*sidebands*) whose relative amplitudes are given by the Bessel functions $J_k(m)$.

Qualitatively describe the changes in the power spectrum of frequency-modulated light as the modulation index goes from small ($m \ll 1$) to large ($m \gg 1$) values.

8.

8.4 Frequency doubling of modulated light

Consider a frequency-doubling device, e.g., a doubling crystal for laser light. Given monochromatic radiation of frequency ω at the input, the output is monochromatic radiation of frequency 2ω with intensity proportional to the square of the input intensity. Suppose now that the input radiation is modulated at a frequency Ω, so its spectrum consists not only of the *carrier* frequency ω, but also of sidebands at combination frequencies.

What is the frequency spectrum (frequencies and relative sizes of spectral components) at the output of the doubler? Assume that the bandwidth of the doubler is large enough to accommodate all the relevant frequency components. Consider the cases of weak amplitude and frequency modulation of the input radiation (i.e., first-order sidebands are much smaller than the carrier, and higher-order sidebands for the case of frequency modulation are negligible).