Due: Wednesday, 9/15/2004

1. Estimate electric field amplitude in a light beam of 1-mm diameter for a) 1 W continuous wave (cw) laser (e.g. a low-power argon ion laser); b) a pulsed laser with pulse energy 1 J and duration 10 ns (a typical Nd-YAG laser). Compare these values to a typical intra-atomic electric field.
2. In a nonlinear optics course, we study nonlinear light interactions in media. However, nonlinear interactions can occur even in vacuum. An example is photon-photon scattering. In quantum electrodynamics it is shown that the cross-section of this process is:

$$\sigma \approx 0.031\alpha^2 r_e^2 \left(\frac{\hbar\omega}{mc^2}\right)^6, \qquad \hbar\omega << mc^2 \ ,$$

where α=e$^2/\hbar c$ is the fine structure constant, r_e=e$^2/mc^2$ is the classical radius of electron, m is the electron mass and ω is the photon angular frequency.

a. Comment on the physical origin of this process. How can one reconcile this effect with linearity of Maxwell's equations in vacuum?
b. Suppose we arrange two P=1 W cw laser beams ($\lambda \approx 0.5\ \mu$) with diameters 1 mm in such a way that they propagate in vacuum in nearly-opposite directions and intersect, so that the intersection length is $l \approx 1$ cm. Estimate scattering probability, i.e. what fraction of photons will be removed from a beam due to the nonlinear interaction.
c. Repeat this estimate for 'colliding' light pulses with duration τ=100 fs (1 fs=10^{-15} s), I=10^{18} W/cm^2. These pulse parameters can be achieved with modern lasers.

3. Consider a model of solid nonlinear medium (e.g. a second harmonic generation crystal) in which individual atoms can be thought of as electrons attached to springs that show deviations from Hook's law at relatively large deviations from an equilibrium position (anharmonic oscillator model). Based on this model,
a. estimate a typical value of the second-order nonlinear susceptibility $\chi^{(2)}$. Compare your numerical result (in esu and/or pm/V) with actual data for nonlinear crystals;
b. show that $\chi^{(2)}$=0 if the medium has a center of inversion, i.e. the potential of an electron is symmetric with respect to the equilibrium position.

4. Show that linear electrooptic (Pockels) effect is absent for crystals with inversion symmetry.

5. Atomic vapor with number density 10^{16} atoms/cm^3 is composed of initially unpolarized atoms in the ground state with J=1 and a Lande factor g=1.5. The vapor is optically pumped with circularly polarized light tuned to a transition to a J=0 excited state, which spontaneously decays back to the ground state. Ignoring relaxation between the ground state Zeeman sublevels, determine the steady-state magnetic field in the sample (inverse Faraday Effect).