Topological Insulators

Yize Jin, Lu Zheng

Department of Physics, Fudan University, China

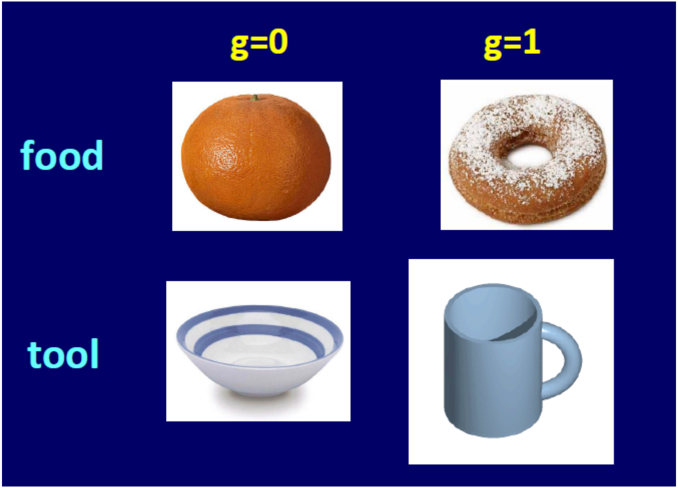
Outline

Outline

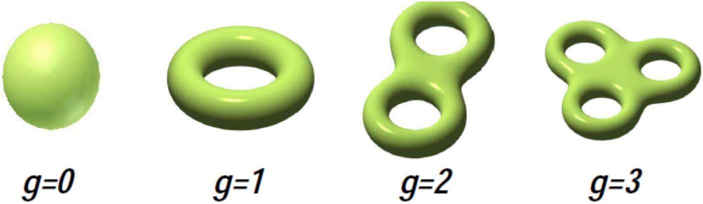
Outline

Outline

Topology



Genus



Topological Insulators

Conductor

Insulator

Topological insulator is insulator in bulk but conductor only on edge.

Quantum Hall Effect

- $|u(\vec{k})\rangle$ is an eigenstate of the Hamiltonian, the Berry's phase of this is $A_m = i\langle u_m|\nabla_k|u_m\rangle$ this may be expressed as a surface integral of the Berry flux

$$F_m = \nabla \times A_m$$

- The first chern number of each state of an electron

$$n_m = \frac{1}{2\pi}\int d^2\vec{k}F_m$$

- The first chern number of an electron

$$n = \sum_m n_m$$

- Hall conductivity

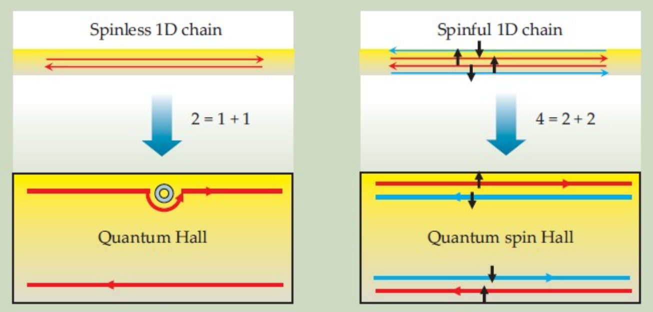
$$\sigma_{xy} = Ne^2/\hbar$$

Outline

Quantum Spin Hall Effect

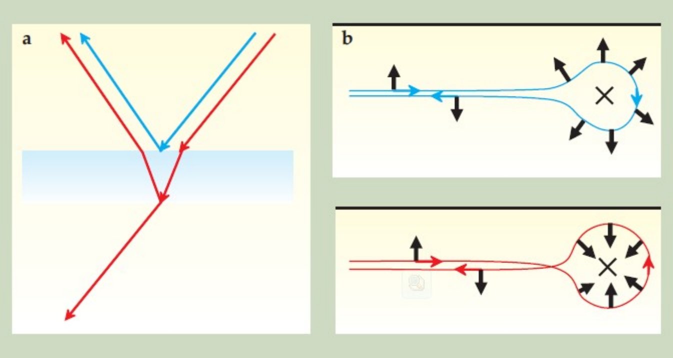
- The conditions of QHE(Quantum Hall Effect) are strong magnetic field and low temperature ,which are hard too realize.
- QHE $\Longrightarrow$ QSHE(Quantum Spin Hall Effect):no magnetic field

Quantum Spin Hall Effect



Electronic current $\Longrightarrow$ Spin current

Quantum Spin Hall Effect



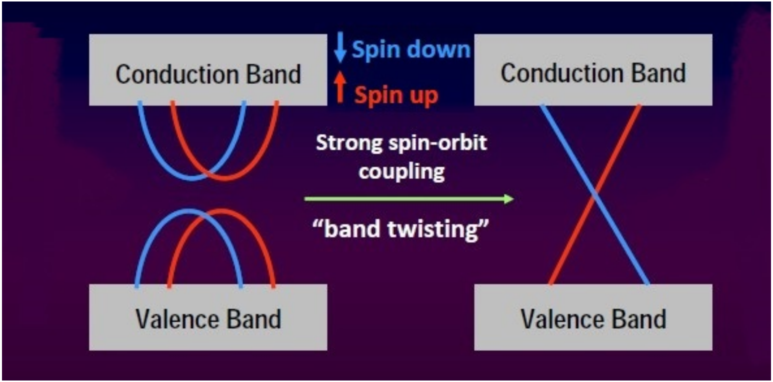
Edge-states electrons in QSHE are immune to impurity scattering

Quantum Spin Hall Effect

- In figure (a), reflected light from upper edge and bottom edge interfere with each other destructively.
- In figure (b),the upper electron is scattered clockwise(π) while the lower counterclockwise($-\pi$).
- Since an electron is a spin-1/2 particle, a $2\pi(=\pi-(-\pi))$ rotation difference will cause a phase difference of -1, resulting in destructive interference.

Outline

Band Structure



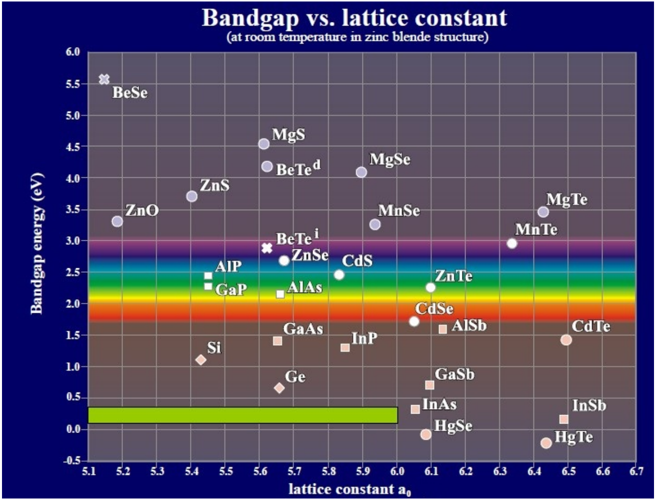
Topologically Inequivalent

Band Structure

- Why are they topologically inequivalent?$\Longrightarrow$ The intersections of trivial insulators can be seperated, but not topological insulators.
- Why can't the intersections of topological insulators be seperated?$\Longrightarrow$ Kramers theorem
- What is Kramers theorem?$\Longrightarrow$The energy levels of systems with an odd total number of fermions remain at least doubly degenerate in the presence of purely electric fields.
- In topological insulators, the red line(edge states) doesn't come back to valence band like trivial insulators. $\Longrightarrow$Band inversion.

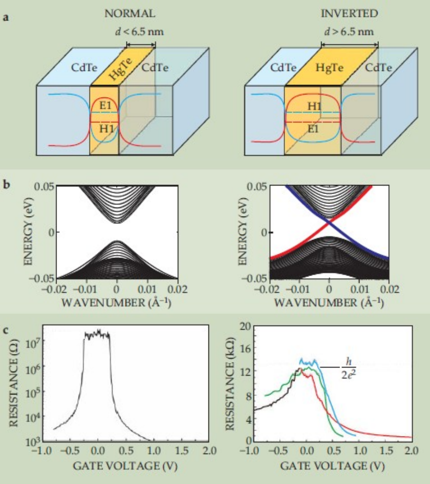
Outline

Forecast


Bandgap vs. lattice constant
(at room temperature in zinc blende structure)
Bandgap energy (eV)
lattice constant a_0
6.0
5.5
5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
-0.5
5.1
5.2
5.3
5.4
5.5
5.6
5.7
5.8
5.9
6.0
6.1
6.2
6.3
6.4
6.5
6.6
6.7
BeSe
MgS
BeTe d
MgSe
ZnS
ZnO
MnSe
MgTe
MnTe
BeTe i
ZnSe
CdS
AlP
GaP
AlAs
ZnTe
CdSe
AlSb
CdTe
GaAs
InP
Si
Ge
GaSb
InAs
HgSe
InSb
HgTe

- HgTe: $E_g < 0 \iff$ p orbital band is above the s orbital band.
- CdTe: $E_g > 0 \iff$ s orbital band is above the p orbital band.
- Make a sandwich$\Longrightarrow$band inversion$\Longrightarrow$topological insulator?

The First Found Topological Insulator



- E1 is the s-like conduction subband and H1 is p-like valence subband.
- $d_c = 6.5nm$
- Thick quantum well has a quantized resistance plateau at

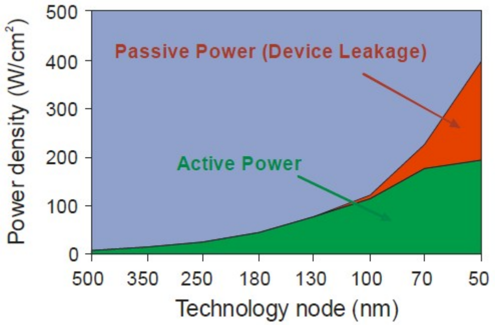
$$R = \frac{h}{2e^2}$$

due to the conducting edge states

Possible Application

- Superconductor
- Topological Quantum Computation
- Anomalous Quantum Hall Effect
- Majorana Fermion
- ……

Spintronic Devices



Impurity scattering $\Longrightarrow$ Heat dissipation

Solution:QSHE

References

- X.-L Qi,S.-C Zhang, *Phys. Today* **63**(1), 33(2010).
- B.A. Bernevig, T. L. Hughes, S.-C. Zhang, *Science* **314**, 1757(2006).
- M.König et al., *Science* **318**, 766 (2007).
- M.Z.Hasan, and C.L.Kane (2010), e-print arXiv:1002.3895.

THANK YOU!