The Fermi Surface of Copper

Shawn Tang
Mar 19 2013
What is a Fermi surface?

- Few people would define a metal as a “solid with a Fermi surface.” This may nevertheless be the most meaningful definition of a metal one can give today… [it] provides a precise explanation of the main physical properties of metals. –A. R. Mackintosh

- The Fermi surface is the surface of constant energy $\epsilon_F$ in $k$ space.

- What does this mean?
Bloch’s Theorem

\[ U(r + R) = U(r) \]

\[ H\psi = \left( -\frac{\hbar^2}{2m} \nabla^2 + U(r) \right) \psi = \varepsilon \psi \]

\[ \psi_{nk}(r) = e^{ik \cdot r} u_{nk}(r) \]

\[ u_{nk}(r + R) = u_{nk}(r) \]

\[ \psi(r + R) = e^{ik \cdot R} \psi(r) \]
From Bloch’s theorem, the index $n$ indicates that for each $k$ there are many solutions

Energy levels

$\varepsilon_n(k)$

Consider instead to allow $k$ to range (rather than holding it fixed)

Eigenstates and eigenfunctions for $k$ that differ by a reciprocal lattice vector are identical due to periodicity

$$\psi_{n,k+K}(r) = \psi_{nk}(r)$$

$$\varepsilon_{n,k+K} = \varepsilon_{nk}$$
We construct a ground state of $N$ Bloch electrons which results in two distinct configurations.

The first configuration involves a number of bands completely filled, with all others empty.
The second configuration involves a number of bands being partially filled.

\( \epsilon_F \) lies within the energy range of at least one band.

For each partially filled band there is a surface in \( k \)-space separating the filled and unfilled levels.

This is the Fermi surface.
Sir Alfred Brian Pippard

- A British physicist who worked on radar during World War II
- Mastering and developing radar techniques
- Understanding radio propagation and microwave components
- Gave him an understanding of the *skin effect* – the restriction of microwave fields to a thin surface layer when EM waves are incident on a metal surface
The Anomalous Skin Effect

- Measuring reflection and absorption of microwave electromagnetic radiation

- Typically the field will penetrate into the metal a distance $\delta_0$ known as the "classical skin depth"
  - $\delta_0 \gg l$ (mean free path)

- But in the other limit $\delta_0 \ll l$ known as the "extreme anomalous regime" we observe different interactions
Use of the anomalous skin effect to measure surface resistance of copper at low temperatures.

At the extreme anomalous regime the surface resistance has a relationship with the curvature of the Fermi surface.

Measuring surface resistance in different orientations to determine geometry of the Fermi surface.

High-purity copper to ensure the extreme anomalous regime.
The Fermi Surface of Copper
References