Due: Thursday, 02/03

1. This problem gives exercises in atomic units, which are discussed in class.
a) What is the Bohr magneton in atomic units?
b) What is the atomic unit of electric field in Volts/cm?
c) What is the atomic unit of magnetic field in Gauss?
d) What are the ground state binding energies of positronium, muonium, atomic deuterium, singly ionized helium-four, in atomic units?

2. a) Use the uncertainty principle to estimate the radius of a hydrogen atom in the ground state. What is the characteristic value of the electron velocity in an atom in units of the speed of light? Is the electron relativistic?
b) Explain why the existence of a stationary planetary atom (like e.g. hydrogen in the ground state) contradicts classical physics. Use the formulae for the radiation of accelerating charge to estimate the time it would take a classical atom to collapse. How does this time scale compare to the electron revolution period?

3. For hydrogen-like ions with nuclear charge Z, find the scaling with Z of the expectation values of the following operators: r^n for n = 1,-1 and -3; the potential energy V; and the total energy E. Also find the scaling with Z of $|\psi(r=0)|^2$ and $\left|\frac{\partial\psi}{\partial r}(r=0)\right|^2$. (Hint: you shouldn't have to use any explicit wavefunctions; just consider the dimensions of the quantities of interest.)

4. Muon capture is a weak-interaction process in which a negative muon is captured by a nucleus (Z,N) with Z protons and N neutrons, resulting in a nucleus (Z-1,N) and a muon neutrino:

$$\mu^- + (Z,N) \rightarrow (Z-1,N) + \nu_\mu$$

This occurs as follows: when negative muons pass through matter, they slow down and some ultimately become bound in atomic orbits forming the so-called mu-mesic atoms. A negative muon in a mu-mesic atom quickly cascades down to a 1s hydrogenic orbital, from which it is captured by a proton in the nucleus. This weak-interaction process, like all others, can only occur when the initial reactants are at very close (essentially 'zero') range; in other words the muon and a proton in the nucleus must be in contact. Assume that the nuclear electric charge is uniformly distributed in a sphere of radius:

$$R = 1.2 \cdot 10^{-13} A^{1/3} \text{cm}.$$

a) At approximately what value of Z (call it Z_0) is the nuclear radius equal to the average distance of the muon from the origin?
b) How does the probability for muon capture W scale with Z for $Z \ll Z_0$?
c) Comment on how the probability for muon capture W scales with Z for $Z \gtrsim Z_0$. Note: this may not be an easy question.

For those who want the whole story, a corresponding detailed discussion can be found e.g. in M. Morita. Beta Decay and Muon Capture. Benjamin, 1973. QC 795 M631