Homework # 2; due Thursday, Sept. 16

Reading: Chapters 2,3 of Griffiths, Wikipedia.

6. *Positronium* (Ps) is an atom that consists of an electron and its anti-particle a positron.
   (a) What can you say about the energy-level structure of positronium? Specifically, discuss the similarities and differences with the case of hydrogen.
   (b) Since both electron and positron are spin-1/2 particles, the total spin of positronium can be either $S = 0$ (*para*-positronium) or $S = 1$ (*ortho*-positronium). Which of these has lower ground-state energy, ortho- or para-positronium?
   (c) Both ground-state ortho- and para-positronium can decay via annihilation of their constituents. However, it turns out that the lifetimes of these two states differ by three orders of magnitude. Shorter-lived para-positronium annihilates emitting two photons, but this process is forbidden for ortho-positronium, so when it annihilates, three photons are emitted. Here we will be concerned with two-photon annihilation of para-positronium. Give an order-of-magnitude estimate of para-positronium lifetime, which is determined by this process.
   Hint: Use dimensional analysis and the fact that an amplitude of a process where a photon couples to an electron or a positron should be proportional to the coupling constant of the electromagnetic interaction, i.e., to the magnitude of the electron (positron) charge, e.

7. Griffiths, Problem 1.19

8.
   a) Griffiths, Problem 2.1.
   b) Now, also roughly (i.e., within ±5 orders of magnitude) estimate the number of particles in the Universe. This can be done, for example, by assuming that the total energy (total mass plus the gravitational energy) of the Universe is zero, and dividing the total mass obtained from this by the mass of a proton. For this estimate, use the fact that the age of the Universe is some 13 billion years.
   c) Compare the two numbers. Dirac thought that the relation between these numbers is not an accident, which came to be known as the *Dirac large-number hypothesis*. Explain how temporal variation of fundamental "constants" follows from such a hypothesis, and estimate, to within several orders of magnitude, the relative variation of constants per year.
   d) Do a quick search to see if such variation is consistent with observations.

9. Griffiths, Problems 2.2 and 2.6

10. Griffiths, Problem 2.7