Introduction to Nuclear Physics

Physics 124

Solution Set 4

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January 3, 2000

Problem 14

In making a "back of the envelope" calculation we must simplify the existing theory and make appropriate assumptions. The transition rate for gamma decay is given by, Krane equation 10.10,

$$\lambda(\sigma L) = \frac{2(L+1)}{\epsilon_o \hbar L[(2L+1)!!]^2}\left(\frac{\omega}{c}\right)^{2L+1}[m_{fi}(\sigma L)]^2. \tag{1}$$

We are interested in electric quadrupole transitions therefore, L = 2. Assuming the transition is due to one nucleon changing from one state to another in the shell model we can simplify the calculation. First of all the spherical harmonic for quadrupole radiation is,

$$e(3z^2 - r^2). \tag{2}$$

We can then calculate the radial part of the transition probability. Assuming the wavefunction of the initial and final state are constant within the nuclear radius and zero elswhere,

$$\frac{\int_0^R r^2 r^L dr}{\int_0^R r^2 dr} = \frac{3}{L+3}R^L. \tag{3}$$

The other crucial approxiamtion is to say that the integral over spherical harmonics gives a result on the order of unity. Now we can see that

$$[m_{fi}(\sigma L)]^2 = [m_{fi}(EL)]^2 = \left(\frac{e3}{L+3}\right)^2 R^{2L} \tag{4}$$

Using L = 2 our final equation is then

$$\lambda(E2) = \frac{2(3)}{\epsilon_o \hbar 2[(2(2)+1)!!]^2}\left(\frac{E}{\hbar c}\right)^5\left(\frac{e3}{5}\right)^2 R^4. \tag{5}$$

We can use R = R_o $A^{\frac{1}{3}}$ and after some algebra we obtain,

$$\lambda(E2) \approx 10^8 E^5 A^{\frac{4}{3}}, \tag{6}$$

where $\lambda(E2)$ is in sec^{-1} and E is in MeV. For a nucleus with A = 50 and a gamma ray of 2 MeV the rate is $5 \times 10^{11}\ sec^{-1}$.

Problem 15

All quantities are calculated in femtometers for comparison.

a) Nuclear radius, using A = 50 as an example,

$$r = 1.2fm \times A^{\frac{1}{3}} \approx 5fm. \tag{7}$$

b) Compton wavelength of an electron,

$$\lambda_{Compton} = \frac{h}{mc} = \frac{6.626 \times 10^{-34} J - sec}{(9.109 \times 10^{-31} kg)(2.9979 \times 10^{8} \frac{m}{s})} = 243fm. \tag{8}$$

c) Reduced photon wavelength for E_γ = 1 MeV,

$$\frac{\lambda_\gamma}{2\pi} = \frac{hc}{2\pi E_\gamma} = \frac{(6.626 \times 10^{-34} J - sec)(2.9979 \times 10^{8} \frac{m}{s})}{2\pi(1MeV)(1.602 \times 10^{-13} \frac{J}{MeV})} = 197fm. \tag{9}$$

d) The de Broglie wavelength of an electron with kinetic energy T_e = 1 MeV. Note first of all that the kinetic energy is related to the momentum by:

$$p = \frac{\sqrt{T_e^2 - m^2c^4}}{c}. \tag{10}$$

Substituting into de Broglie's equation for wavelength,

$$\lambda = \frac{h}{p} = \frac{hc}{\sqrt{T_e^2 - m^2c^4}} = \frac{(4.136 \times 10^{-21} MeV - sec)(2.9979 \times 10^{8} \frac{m}{s})}{\sqrt{(1MeV)^2 - (0.511MeV)^2}} = 1443fm. \tag{11}$$

e) A typical de Broglie wavelength of an α particle. Using an α particle with kinetic energy T_e = 4 MeV for example we find,

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mT_e}} = \frac{6.626 \times 10^{-34} J - sec}{\sqrt{2(6.6466 \times 10^{-27} kg)(4MeV)(1.602 \times 10^{-13} \frac{J}{MeV})}} = 7fm. \tag{12}$$

Observations: The more massive the particle, the shorter the wavelength. The more energetic the particle is, the shorter the wavelength is. Which is expected since $E^2 = p^2c^2 + m^2c^4$.

Problem 16

All information for this problem can be found in the Table of Isotopes.
a) The excited state of ^{137}Ba is $J^\pi = \frac{11^-}{2}$ while the ground state is $J^\pi = \frac{3^+}{2}$. Using angular momentum selection rule we can see that the allowed multipolarity are L = 4,5,6, and 7. The parity also changes in this decay, and thus we know that electric transitions must be odd L and magnetic transitions are even L from the rules

$$\pi(EL) = (-1)^L \quad and \quad \pi(ML) = (-1)^{L+1}. \tag{13}$$

Summarizing we can say that we have the following transitions; M4, E5, M6, and E7.
Note that M4 radiation is 100 less likely than E4 radiation. Also each increase in multipole order corresponds to a reduction in strength by a factor of approximately 10^{-5}. Therefore the M4 transition is a factor of 1000 times more likely than the E5 transition and is the dominant multipole. b) Using the internal conversion plots for Z = 50 we can find the following internal conversion coefficients for the M4 transition;
K-shell = 0.09
L_1-shell = 0.01
L_2-shell = 0.002
L_3-shell = 0.0015
Adding up these results we get a total internal conversion coefficient $\lambda_e =$

0.1035. Therefore the branching ratio for internal conversion in $^{137}Ba^* \rightarrow$ ^{137}Ba is 0.1035. Experimentally this branching ratio is 0.0916 from the Table of Isotopes. So we are off by about 10 % by using the theoretical curves for Z = 50.

Problem 17

We can determine the selection rules by calculating the matirx elements using the two operators defined in the problem.
The Fermi operator is used to find,

$$< T_f T_{of} J_f M_f | O_F | T_i T_{oi} J_i M_i > \tag{14}$$

$$=< T_f T_{of} J_f M_f | G_V \sum_{j=1}^{A} t_{\mp}(j) | T_i T_{oi} J_i M_i > \tag{15}$$

$$= G_V \sqrt{T_i(T_i+1) - T_{oi}(T_{oi} \mp 1)} \delta_{J_f J_i} \delta_{M_f M_i} \delta_{T_f T_i} \delta_{T_{of}(T_{oi} \mp 1)}. \tag{16}$$

Examining this equation we find that

$$J_f = J_i \quad (\Delta J = 0), \tag{17}$$

$$T_f = T_i \neq 0 \quad (\Delta T = 0, but T_i = 0 \rightarrow T_f = 0 forbidden), \tag{18}$$

$$T_{0f} = T_{0i} \mp 1 \quad (\Delta T_0 = 1). \tag{19}$$

Now examining the Gamow-Teller operator we have,

$$< T_f T_{of} J_f M_f | O_{G-T} | T_i T_{oi} J_i M_i > \tag{20}$$

$$=< T_f T_{of} J_f M_f | G_A \sum_{j=1}^{A} \sigma(j) t_{\mp}(j) | T_i T_{oi} J_i M_i > \tag{21}$$

We arrive at similar rules except that the spin component allows for more possible transitions,

$$\Delta J = 0, 1 \quad but J_i = 0 \rightarrow J_f = 0 forbidden, \tag{22}$$

$$\Delta T = 0, 1 \quad but T_i = 0 \rightarrow T_f = 0 forbidden, \tag{23}$$

$$T_{0f} = T_{0i} \mp 1 \quad (\Delta T_0 = 1). \tag{24}$$

The reader is referred to section 5.6 of Introductory Nuclear Physics by Samuel Wong for a more detailed discussion.